The FOWLER "Magnum" Long Scale CALCULATOR

INSTRUCTIONS

With numerous Examples

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Price 1/6

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FOWLER S "MAGNUM" CALCULATOR

Fowler's "Magnum" Calculator, like the well-known waistcoat pocket instrument consists of a series of concentric circular scales, logarithmically divided and mounted on a dial capable of rotation by a thumb nut outside the containing case The scales are equipped with a fixed radial datum line, and a radial cursor line rotated by a second thumb nut. The rotating scales cursor line, and operating mechanism are enclosed in a metal containing case, fitted with a glass face so that the scales are always kept clean, and the instrument is preserved from external injury in a handsome wallet which fits easily into a side pocket. The large size of the "Magnum" enables all the scales to be mounted on one dial, and so of being synchronised and read concurrently. It also permits of the use of larger figures and easier reading, an advantage to persons of weak eyesight. Another important feature is that the scales are longer and admit of finer graduation and more accurate reading. This is specially noticeable in the "Long-Scale" which gives a length of 50 ins. as compared with 10 ins. in the ordinary slide-rule and permits of calculations being made to four, and sometimes five, significant figures. The motions of the scales and cursor can be made with great ease and nicety and there is none of the objectionable sticking or slackness often so troublesome with the straight, slide rule, nor is there any of the "end switching" often necessary with that instrument when working with the full length scale and which induces many users to work habitually with the half length. In circular calculators the scales are continuous and there is no half length.

To sum up the merits of Fowler's "Long-Scale" Instruments, they are more portable, comprehensive and accurate; cleaner and easier to operate, and may be used equally well in any climate.

Logarithmic Calculation.—The mathematical basis on which all instruments of this kind rest is that of logarithms, first discovered by Napier, which permit of the tedious arithmetical operations of multiplication and division being replaced by the simpler ones of addition and subtraction, which can be done with two sliding scales having logarithmic distances marked on them as in the straight slide-rule.

Logarithmic graduation, however, is not uniform and such scales, whether straight or circular have certain drawbacks. The intervals which represent logarithmic distances between the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, diminish rapidly from 1 to 10. If the intervals be expressed as percentages of the whole length they are respectively 30, 17·6, 12·5, 9·7, 7·9, 6·7, 5·8, 5·1, 4 5,

and show how much finer the interval at the beginning between 1 and 2 can be graduated than the one at the end between 9 and 10. One is nearly 7 times as great as the other and it is impossible in a straight scale of reasonable length to secure uniform graduation. Some parts are divided by 10, others by 5 and others again by only 2. This dissimilarity of division prevents equal accuracy of reading in all parts of the scale.

A length of less than 10 inches in a straight slide is not of much practical use and is not a very portable article. If the scale, however, is arranged in circular form, round one circle or round several, as in Fowler's "Long-Scale," it may afford a length of 30 ins. and yet be capable of insertion in the waistcoat pocket, while in the case of the "Magnum," which can be carried in the side pocket, the length of the Long Scale is 50 inches. The advantage of this is shown in the following examples.

Description of Scales.—There are seven separate scales. Beginning with the largest in diameter, No. 1, and proceeding inwards to the smallest, they are as follows:—

Scale No. 1.—The "Short-Scale," a single circle, 13½ inches in circumference, graduated clockwise. This is the calculating scale for multiplication, division, etc., analogous to the ordinary slide rule. It is also used for direct reading and incorporating values of functions on other scales by aid of the cursor or datum line (the cursor by preference, as it is closer to the scales and eliminates parallax).

Between the prime numbers **1** and **2** the scale is divided into 10 figured parts (11, 12, 13, ...) each decimally graduated and capable of further graduation with the cursor. Readings on this part can be made easily to four, and sometimes five, significant figures.

Between the prime numbers, **2, 3, 4, 5,** each part is divided into 10, viz., 21, 22, ... up to **3,** 31 ...... up to **4,** 41, 42, ... up to **5,** though only even divisions, 22, 24, 26, etc., are figured.

Between the prime numbers, **5, 6, 7, 8, 9, 10,** each part is divided into 2, viz., 55, 65, ... up to 95, and each of these decimally graduated, so that on this, the finest part of Scale No. 1, readings can be made to three and sometimes four significant figures.

Scale No. 2.—The "Reciprocal Scale," a single circle, exactly like No. 1, but graduated **contra clockwise** so that the readings on one are the reciprocal values of those radially opposite on the other. On this scale the values increase from **right to left** instead of from left to right as in all the other scales. It should be noted that any value on Scale No. 5 (the Long Scale) must be expressed on Scale No. 1 before its reciprocal value can be read on Scale No 2.

Scale No. 3.—The Square Root Scale. A scale extending round the inner and outer circumference of a common circle and giving the square root values of readings on Scale No. 1 which conversely gives the squares of the values on Scale No. 3.

This scale, which has a total length of 22 inches, may, if desired, be also used for multiplying and dividing, though these operations are usually made on No. 1 The Short Scale), or No. 5 (The Long Scale).

Scale No. 4.—A Scale of Logarithms uniformly graduated in 500 divisions from 0·002 to 1·0.

For a given number read on Scale No. 1 the Naperian logarithm is read on Scale No. 4 and conversely for a given Naperian or Common Logarithm read on Scale No. 4, the corresponding number is read on Scale No. 1.

Scale No. 5.—" The Long-Scale." A scale extending round six circles, beginning at the smallest and continuing round the successive circumferences until it completes the sixth, with a total length of 50 inches.

This scale gives the sixth root values of Scale No. 1, which conversely gives the sixth power values of Scale No. 5. It also enables cube roots to be read at a single setting. Since if x is a number:

$$\sqrt[3]{x} = \sqrt[6]{x^2}$$

Therefore, if x on Scale No. 3 is set under cursor the reading on Scale No. 1 is x^2 and the reading on Scale No. 5 is

$$\sqrt[6]{x^2} \ \textit{i.e.}\ \sqrt[3]{x}$$

In a converse way the cubes of numbers may be read directly by setting the number x on Scale No. 5 under the cursor and reading x^3 on Scale No. 3.

A mental estimate of the value of the cube root, or of the cube, is, of course, required to determine on which particular circle of scales Nos. 5 or 3

$$\sqrt[3]{x} \ \textit{or}\ x^3$$

are to be found. This will be discussed when giving examples of the use of the scales.

The most valuable feature of Scale No. 5 is its great length (50 inches) which permits of multiplication and division with a degree of accuracy beyond the possibility of any straight slide rule. The Scale is used just like No. 1, in setting factors, but requires a little mental consideration like any other logarithmic

scale to determine the result and hence the circle on which it is to be read. This is more fully discussed in describing the practical use of the scale. Its great length enables it to be divided into 100 figured parts, 1, 2, 3, 4, ... up to 100, and each of these to be graduated decimally, which makes setting and reading very simple. Three significant figures of a result can be written without hesitation, even in the most finely graduated part between 99 and 100, while further division can be made with the cursor and over a great part of the scale results can be read to four, and sometimes five, significant figures. The superiority of such a scale over that of the 10 inch slide rule will be manifest to those familiar with that instrument which only permits 2 graduations between 99 and 100 as compared with 10 graduations of the Long-Scale of the " Magnum Calculator."

Scale No. 6.—A Scale of Angles giving Natural Sines and Log. Sines, extending round the inner and outer circumferences of a common circle. The inner circle gives angles from 35 minutes to 5 degrees 45 minutes; the outer circle angles from 5 degrees 45 minutes to 90 degrees, graduated as follows :—

From	35 mins.	to 2 degrees	at	intervals of	1 min.
,,	2 deg.	,, 4 ,,		,,	2 ,,
,,	4 deg.	,, 10 ,,		,,	5 ,,
,,	10 deg.	,, 20 ,,		,,	10
,,	20 deg.	,, 35 ,,		,,	20 ,,
,,	35 deg.	,, 50 ,,		,,	30 ,,
,,	50 deg.	,, 70 ,,		,,	1 deg.

Scale No. 7.—A Scale of Angles giving Natural Tangents and Log. Tangents. The Scale gives angles from 5 degrees 45 minutes to 45 degrees graduated as follows :—

From	5 deg. 45 min.	to 10 deg.	at intervals of	5 min
,,	10 deg.	,, 20 deg.	,,	10 ,,
,,	20 deg.	,, 35 deg.	,,	20 ,,
,,	35 deg.	,, 45 deg.	,,	30 ,,

The Natural values of Sines and Tangents are read on Scale No. 1 and Log. values on Scale No. 4.

The dial, with its seven scales, is rotated by the thumb nut at the top of the instrument, and the Cursor line by the thumb nut at the side. The fixed datum line is on the cover glass of the instrument. The unity or zero line is common to all the scales.

Scales No. 1 and No. 5 are provided with a number of useful conversion Factors the values of which are as follows :

Metres to yards, 1·09361 ; yards to metres, 0·9144 ; sq. metres to sq. yards ,1·19599 ; sq. yards to sq. metre 0·83613 ; sq. centimetres to sq. inches, 0·155 ; sq inches to sq. centimetres, 6.45159 ; miles to kilometres 1·60934 ; kilometres to miles, 0·62137 ; kilogrammes to lbs. 2·20462 ; lbs. to kilogrammes, 0·45359 ; inches to centimetres, 2·54; centimetres to inches, 0·3937; sq. miles to sq. kilometres, 2·5899 ; sq. kilometres to sq. miles 0·3861. "C" = 1·12838 and is a constant which when multiplied by the square root of the area of a circle give its diameter. $\sqrt{2}=1\cdot41421$; $\sqrt{3}=1\cdot73205$; $\text{Log.}_e10=2\cdot30258$; $\text{Log.}_{10}e=0\cdot43429$; $\pi=3\cdot14159$; g_E = 32·2 feet per second ; $g_F=9\cdot81$ metres per second radian = 57·2958 degrees ; E.H.P. = 746 ; $\pi/4=0\cdot7854$

The difficulty experienced by a beginner in using logarithmic scales of any kind for calculation is largely due to the variable nature of their graduations and of the values attached to them not only to the graduation lines themselves but to the spaces between them and which must be allowed for when setting values and reading results. The spaces between the prime numbers 1, 2, 3, 4, ... 10 differ greatly and the numbers themselves may represent their simple values or any multiple of 10 thereof, while the fine graduations of the scale may be of the nature of 2, 5, or 10. These features are at first confusing, but when the learner is familiar with them he will find difficulties disappear and that the calculations can be made with confidence and accuracy as well as great saving of labour, and his attention should be first directed to this end. As he acquires facility in the use of the scales he will discover many short cuts in manipulating them and need not strictly follow the instructions here given for his guidance in the form of a number of worked out examples which he is recommended to study in detail, remembering that the Calculator is a tool requiring for its efficient use the exercise of a little common sense and mental arithmetic. It is not an adding machine for totalling money fractions, like a bank clerk, but a device for making rapidly and with practical accuracy the innumerable calculations required by designers, engineers, chemists, draughtsmen, and students in the course of their daily work.

An Additional Constant.—A constant which is very useful is that giving the area of a circle of one inch diameter, expressed in square feet, viz. :—0·785398/144, or 0·00545415. This number, multiplied by A^2, gives at once, in square feet, the area of a circle of diameter A inches.

Multiplication and Division.—The great bulk of calculating work consists of multiplication and division. These operations are in essence addition and subtraction, even in ordinary arithmetic and with the Calculator are reduced to these elementary principles. The working out of a compound fraction, for instance, containing several fractions in the numerator and in the denominator resolves itself into rotating the added numerator factors in one direction, and the subtracted denominator factors in another direction.

Just as there are several ways of working out a compound fraction sum by arithmetic, there are several ways of doing it with a calculator. The numerator factors may be all multiplied together and divided by the total product of the denominator factors, or the factors of the numerator and denominator may be dealt with in pairs, one after the other. Sometimes one method is better than another. The user will discover best methods and short cuts for himself as he acquires proficiency. His first step is to master the principles of operation by studying a few practical examples worked out and described in detail.

Ex. 1 : *Find the product of the factors a, b, c, d,*

Either the Short Scale, No. 1, or the Long Scale, No. 5, may be used. For this illustration Scale No. 1 will be used.

Set factor *a* under datum.
Set cursor to one.
Set dial till factor *b* comes under cursor.
Read product $a \times b$ under datum.
Next set cursor again to one.
Set dial till factor *c* comes under cursor.
Read product $a \times b \times c$ under datum.
Again set cursor to one.
Set dial till factor *d* comes under cursor.
Read product $a \times b \times c \times d$ under datum.

The process after setting the first factor a under the datum is a succession of settings of cursor and of factors on the scale, and of finally reading the product under the datum. The whole operation begins at the datum and ends there.

If there are decimal points in the factors the position of the point in the final product is to be decided by inspection and mental consideration as with all logarithmic work. Actual examples of this will be given in the course of the exercises.

If Scale No. 5 (Long Scale) is used instead of Scale No. 1 (Short Scale) the succession of operations is precisely the same, but the setting calls for a little more care as the factors are spread over a scale extending round six circles, and the answer may also be on **any** one. The particular circle must be determined **by a** mental consideration of the factors, or it may be determined by first roughly and rapidly working out the problem on the short scale, no special care being taken in setting the factors to either datum or cursor. An approximate answer will then result, but one sufficiently accurate to give the *location* of it, when worked out on the long scale, with care taken in all settings. Such a quick trial on the short scale can be made in less time than it takes to describe, but it eliminates any reading from a wrong circle when the long scale is used.

When doing tabular work or working out a series of scientific results the location of the first often serves as a guide to the location of the following ones, the Long Scale then can often be read just as easily as the Short Scale.

To get accustomed to reading Scales Nos. 1 and 5 and their graduations the learner will find it good practice to work through a multiplication table such as twice one are two, twice two are four, etc., thus :
Set 2 on Scale 1 under datum and set cursor to unity
Then turn, in succession, all the figured graduations past the cursor, noting that the procession of values which pass the datum are twice those which pass the cursor.

Thus $2 \times 11 = 22$; $2 \times 12 = 24$; etc

Do the same for 3, or other simple number and proceed to such multipliers as 3·1, etc.

This kind of exercise teaches the learner to read accurately parts of the scale that are not figured or where the graduations require to be split in reading and each counted as 2 if there are 5 graduations, or each counted as 5 if there are 2 graduations.

Division.—This is in essence subtraction, and the reverse of multiplication, which is addition. Its performance with the Calculator is best acquired by practising with simple fractions till the routine of operations becomes mechanical.
Assume the division is of a simple fraction form $\frac{a}{m}$ i.e., with a single numerator and a single denominator, and also that Scale No. 1 is being used, and the learner is advised to get accustomed to the Scales Nos. 1 and 2 before using Scale No. 5.

Set a under datum.
Set cursor to m.
Set one to cursor.

Read value of $\frac{a}{m}$ under datum.

Scale No. 2 (the Reciprocal Scale graduated anti-clockwise) is often useful for many fractions of the class $\frac{a}{f(x)}$ where the denominator may be x^2, sin. x, tan. x, etc., or any other function read directly on Scale No. 1 and whose reciprocal is at the same time given on Scale No. 2.
Such a fraction then becomes simply the **multiplication** of two factors, viz., $\frac{1}{f(x)}$ and a, and can be done at two settings, but $\frac{1}{f(x)}$ must be set first and then multiplied by the factor (or factors if there are more than one) in the numerator.
This is only one of many devices that can be adopted with the Calculator and make it superior to the straight slide rule.
If the fraction is of one of the following forms :—

$$\frac{a \times b \times c}{m} \qquad \frac{a \times b}{m \times n} \qquad \frac{a \times b}{m \times n \times p}$$

where the numerator has not exactly one factor more in it than the denominator it can best be worked by using up a factor from the top and bottom alternately, and to adopt this method and prevent confusion in operating, the numerator should always contain one more factor than the denominator and to secure this, the artifice of inserting a factor (1) is adopted as often as may be necessary. The above fractions, therefore, before using the Calculator are best changed to the following :—

$$\frac{a \times b \times c}{m \times 1} \qquad \frac{a \times b \times 1}{m \times n} \qquad \frac{a \times b \times 1 \times 1}{m \times n \times p}$$

and worked as follows :—

Taking the fraction $\frac{a \times b \times c}{m \times 1}$

Set factor a under datum.
Set cursor to m.
Set factor b to cursor.

Set cursor to 1.
Set factor *c* to cursor.
Read answer under datum.

Taking the fraction $\frac{a \times b \times 1}{m \times n}$

**Set factor *a* under datum.
Set cursor to *m*.
Set factor *b* to cursor.
Set cursor to *n*.
Set factor 1 to cursor.
Read answer under datum.**

Taking the fraction $\frac{a \times b \times 1 \times 1}{m \times n \times p}$

Set factor *a* under datum.
Set cursor to *m*.
Set factor *b* to cursor.
Set cursor to *n*.
Set factor 1 to cursor.
Set cursor to *p*.
Set 1 to cursor.
Read answer under datum.
It will be observed in all these three examples :—
The factors are taken alternately from the numerator and the denominator beginning with the numerator.
The dial is always turned for multipliers.
The cursor is always turned for divisors.
The datum is always used to set first factor and to read final result.

PRACTICAL WORKED OUT EXAMPLES

Multiplication of Two Factors on Short Scale No. 1

Ex. 2 : *Multiply* 12·8 *by* 5·62.
Set 12·8 on Scale No. 1 under datum.
This is the 8th graduation past the 12.
Set cursor to unity line.
Set dial till 5·62 on Scale No. 1 comes under cursor.
This is between the 55 and 6 mark; the exact point being 2 divisions past the 55 to make the 56 and two fifths of the next division to make the 2 of 562.
Read answer just under 72 on Scale No. 1 under datum
We should estimate this as 71·9.
The exact answer by ordinary multiplication is 71 936

Multiplication of the same Two Factors on the Long Scale No. 5.

Set 12·8 on the 1st circle of the long scale from the centre, under datum.

Set cursor to Unity line.
Set dial till 5 62 on the 5th circle from the centre of the long scale comes under cursor.
This is the 2nd graduation past the 56 mark.
Read answer approximately 7195 on Circle No.6 from centre.
We should estimate this as a little under 7194, and perhaps give it a value of 71.935, which would be nearly exact.
*Ex.*3 : *Multiply* .0347 *by* 2·8 *on the Short Scale No.* 1
Set 347 on Scale No 1 under datum.
This lies between the 34 and 36 marks; the exact point being three and a half sub-divisions past the 34 mark.
Set the cursor to Unity line.
Set dial till 28 comes under cursor.
Read answer (just over 97 and which we judge as 972) on Scale No. 1 under datum.
By visual inspection it will be seen that the answer must be in the neighbourhood of ·09. Therefore we write our answer as given by the Calculator as ·0972
Worked out on the Long Scale the procedure is as follows :—
Set 347 the fourteenth sub-division past the 34 under the datum.
Set cursor to Unity line.
Set dial till 28 (3rd circle from centre) comes under cursor.
Read answer a shade over one and a half divisions past the 97 mark.
This we should estimate as 9716, and from the considerations mentioned when worked out on the Short Scale, we should call it 0·09716, an answer correct to five figures.

Multiplication of Three Factors on the Short or Long Scale

The method is precisely the same whichever scale is used, so it will be described only for the Short Scale.
Ex. 4 : *Find the product of* .0347×2·8×63·5
Set 347 on Scale No. 1 under datum.
Set cursor to Unity line.
Set dial till 28 on Scale No. 1 comes under cursor.
All the above settings as shown in *Ex.* 3
Set cursor to Gnity line.
Set dial till 635 on Scale No. 1 comes under cursor.
This is the 7th division past the 6 mark.
Read answer 6·17 on Scale No. 1 under datum.
By actual multiplication the correct answer is 6·16966, showing a close approximation by the use of the Short Scale of the instrument.

Had the Long Scale been used we should have found the answer to come just a *shade* under 617 mark and put the answer down as 6·1695, a still better approximation.

Multiplication of Four or more Factors on the Short or Long Scale.

Ex. 5 : *Find the product of* ·0347 X 2·8 X 63·5 X 4·9.
The method is precisely the same for the first three factors shown in *Ex.* 4 above when we had a reading of 617 (approx.) under the datum.
We now again set the cursor to Unity and then set dial till 49 comes under the cursor.
Read product ·0347 X 2·8 X 63·5 X 4·9 under datum.
This, if using the Short Scale, comes just a little over the first division past the 3, and we should estimate the answer as 30·23 an approximately exact answer.
Ex. 6 : *Divide* 7256 *by* 13·85.
Set 7256 under the datum.
(On the Short Scale this number would be represented as nearly as possible at a point a little over the 5th graduation mark following the 7. On the Long Scale it would be at a point a little over half way between the 725 and the next short division).
Now set the cursor to 13·85.
On the Short Scale this is at a point midway between the 8th and 9th graduation past the 13. On the Long Scale it is at a point on the 1st circle from the centre midway between the 2nd and 3rd graduation following the 138.
Turn dial till the Unity line comes under the cursor.
Read answer 524 on the Short Scale No. 1 and 523·9. on the Long Scale, this latter figure being quite correct

FRACTIONS

Consider first a fraction with two factors in the numerator and one in the denominator, and worked out on the Short Scalre.

Ex. 7 : Solve $\dfrac{676.9 \text{ X } 364}{114.2}$

Set dial till 6769 comes under datum
Set cursor to 1142.
Set dial till 364 comes under cursor.
Read answer 2157 under datum.
The correct answer by actual multiplication and division is 2157·5. When worked out on the Long Scale the answer came barely 2158 and we should therefore put it down as 2157·5, a correct result.

It will have been noticed in working out the previous examples on the Long Scale, that greater *initial* accuracy of setting can be got than when the Short Scale is used and thus it is that a more accurate result is obtained when reading the answer. The method of locating the circle on which to read the answer which has been given previously should however always be borne in mind. The principle of working is, however precisely the same when either scale is used, and to avoid confusion the succeeding examples will all be worked out on the Short Scale.

Consider now fractions with several factors in the numerator and denominator.

Ex. 8 : *Solve* $\dfrac{19{\cdot}5 \times 66{\cdot}6 \times {\cdot}0042}{8.9}$

Work this as $\dfrac{19{\cdot}5 \times 66{\cdot}6 \times {\cdot}0042}{8{\cdot}9 \times 1}$ taking the factors alternately from the numerator and the denominator
Set 195 under datum.
Set cursor to 89.
Set 666 to cursor.
Set cursor to Unity.
Set 42 to cursor.
Read answer ·613 under datum the decimal point being fixed by a rough mental calculation as indicated in examples which follow.
The correct answer worked out by actual multiplication and division is ·61287.

Ex. 9 : *Solve* $\dfrac{13{\cdot}8 \times 723{\cdot}6}{15{\cdot}8 \times 176 \times 2{\cdot}42}$

Work this as $\dfrac{13{\cdot}8 \times 723{\cdot}6 \times 1 \times 1}{15{\cdot}8 \times 176 \times 2{\cdot}42}$

taking the factors alternately as in the previous example
Set 138 under datum.
Set cursor to 158.
Set 7236 to cursor.
Set cursor to 176.
Set Unity to cursor.
Set cursor to 242.
Set Unity to cursor.
Read answer 1·483 under datum.
The correct answer is 1·48388 (a close approximation)

EXERCISES WITH THE RECIPROCAL SCALE NO. 2

If the reader has followed the previously worked out examples carefully he will be in a position to solve in a routine way any compound fraction presented to him, and also to apply the more rapid method of multiplication and division permitted when Scales Nos. 1 and 2 are used in conjunction and which will now be explained.

Multiplication of an ODD number of Factors using Scales Nos. 1 and 2 in conjunction.

Ex. 10: *Find the product of* 8·42 × 16·16 × ·422 (3 factors).

Set 842 on Scale No. 1 under datum.
Set cursor to 1616 on Scale No. 2.
Set 422 on Scale No. 1 under cursor.
Read answer 57·4 on Scale No. 1 under datum.

By actual multiplication the correct answer is 57·42036. The decimal point is fixed mentally in this way: ·422 is roughly ·5; ·5 × 8·42 is roughly 4, and 4 × 16·16 is roughly 56. Therefore there are two whole numbers in the answer.

Ex. 11 *: Find the product of* ·354 × 29·4 ×63·6 × ·862 (4factors).

This will be worked as ·354 × 29·4 × 63·6 × 862×1 to make it into an odd number of factors, i.e., 5.

Set 354 on Scale No. 1 under datum.
Set cursor to 294 on Scale No. 2.
Set 636 on Scale No. 1 under cursor.
Set cursor to 862 on Scale No. 2.
Set Unity on Scale No. 1 under cursor
Read answer 571 on Scale No. 1 under datum (5 move ments).

The correct answer by actual multiplication is 570·578

The decimal point is fixed mentally in this way: ·354 is roughly one-third; one third of 29.4 is roughly 9: 9 times 63·6 is roughly 560; 560 multiplied by ·8 is roughly 500. Therefore the answer must contain three whole numbers and is 571.

It is interesting to compare the above example with the 9 movements necessary when using either Scale Nos. 1 or 5 alone, or by comparing it with the movements of an ordinary Straight Slide-Rule with its intermittent "end-switching." This is only one of many illustrations that could be given.

Rapid Division with Scales Nos. 1 and 2 used in conjunction, with EVEN number of factors in the Denominator.

Ex. 12 : *Find the value of* $\frac{6734}{9\cdot6\times142\cdot5}$ *where there is an E V E N number of factors in the denominator*

Set 6734 on Scale No. 1 under datum.
Set cursor to 96 on Scale No. 1.
Set 1425 on Scale No. 2 under cursor.
Read answer 4·92 on Scale 1 under datum (3 movements, the position of the decimal point being fixed mentally as explained above. The correct answer worked out by multiplication and division is 4·923.

Ex. 13 : *Solve* $\frac{4276}{3\cdot42\times18\cdot7\times32\cdot62}$

Here the artifice may be adopted of inserting an extra factor, 1, into the denominator to make it contain an *even* number of factors, thus : $\frac{4276}{3\cdot42\times18\cdot7\times32\cdot62\times1}$

Set 4276 on Scale No. 1 under datum.
Set cursor to 342 on Scale No. 1.
Set 187 on Scale No. 2 under cursor.
Set cursor to 3262 on Scale No. 1.
Set 1 (Unity) under cursor.
Read answer 2·050 on Scale No. 1 under datum (5 movements).

Further exercises With the Reciprocal Scale

Ex. 14 : *Find the decimal equivalent of* $\frac{1}{6\cdot456}$

Set cursor over 6456 on Scale No. 1. Read under cursor on Scale No. 2 0·1548

In setting cursor to 6456 on Scale No. 1, we note that there are 20 graduations between 6 and 7, the reading advancing clockwise 6·05, 6·10, 6·15, 6·20, etc. and 6·456 is between 6·4 and 6·5 its exact position being estimated. Conceive this space to be divided into 100 parts, and advance 56 of these parts past 6.4 i.e., just a little more than half-way.

Reading Scale No. 2 anti-clockwise, we make the value under the cursor as near as may be 1548.

From inspection of the fraction its value is obviously between one-sixth and one-seventh and without hesitation write down the decimal value as 0'1548.

Ex. 15 : *Find decimal equivalent of* $\frac{1}{3475}$

Set cursor over 3475 on Scale 1.
Read 2878 on Scale 2. under cursor

The fraction is manifestly less than $\frac{1}{3000}$ and expressed decimally will require 3 cyphers after the decimal point, so we write the answer 0'0002878.

In setting 3475 under the cursor we note it falls between the graduations 34 and 35, and that between 34 and 35 there are 5 graduations, each advancing 2, thus: 340, 342, 344, 346, 348, 350. Half-way between 346 and 348 is 347 and a shade past this is 3475.

Reading scale No. 2 the cursor is just short of the value 288. We estimate it as 2878 and the answer, therefore, as 0'0002878.

Ex. 16 : *Find the decimal equivalent of* $\frac{1}{0{\cdot}0284}$

Set cursor over 284 on Scale No. 1. (This is the second graduation line past the 28 mark.

The reading on Scale No. 2 under cursor is just past the graduation the following 35 mark, reading anti-clock wise, and where each space counts 2. We estimate it as 3521.

By inspection the value of the fraction is seen to be more than $\frac{1}{30}$ and we write down the answer as 35·21.

The three preceding examples are good illustrations of the care required in scale reading; in noting the value of the graduations, and whether the advance of the scale is clockwise or anti-clockwise. Scale No. 2. the Reciprocal Scale, it may be noted, is the only one graduated anti-clockwise.

Ex. 17 : *Find the decimal value of* $\frac{1}{37}$

Set 37 on Scale No. 1 under cursor.
Read .027 on Scale No. 2 under cursor.

Note that in reading decimal values of fractions less than one-tenth there will be one cypher placed after the decimal point, and preceding the number as read from the Reciprocal Scale. With values less than one-hundredth and greater than one-thousandth two cyphers will precede the number and so on.

Ex. 18 : *Find the fractional value of* .1428
Set (anti-clockwise) 1428 on Scale No. 2 under cursor.
Read 7 on Scale No. 1 under cursor.
Fractional value is therefore $\frac{1}{7}$.

Ex. 19 : *Find the fractional value of* .00653
Set 653 on Scale No. 2 under cursor.
Read 153 on Scale No. 1 under cursor.
Fractional value is therefore $\frac{1}{15300}$

Note that as many cyphers must follow the 153 as there are cyphers following the decimal point in the given number.

Examples of Powers (Involution) using Reciprocal Scale.

Ex. 20 : *Find value of* $(36.7)^2$

This can be done with the Calculator in two ways, either by multiplying 36.7 by itself as an ordinary multiplication sum, as previously described, or by the method shown below using the Reciprocal Scale.

Set 36.7 on Scale No. 1 under datum.
Set cursor to 36.7 on Scale No. 2.
Turn dial till 1 comes under cursor.
Read 1347 under datum on Scale No. 1.

Ex. 21 : *Find the value of* $(16.4)^3$

This can be done by extended multiplication, 16.4 × 16.4 × 16.4 on Scale No. 1, or by the method shown below, in which we first find the square of 16.4, as in Example above, and then multiply this result on Scale No. 1 by 16.4.

Thus set 16.4 on Scale No. 1 under datum.
Set cursor to 16.4 on Scale No. 2
Turn dial till 16.4 on Scale No. 1 comes under cursor.
Read 4410 under datum on Scale No. 1.
Note—The result is obtained in 3 movements.

The use of the Reciprocal Scale also makes possible the calculation of expressions such as $(.310)^{2{\cdot}1}$ or $(.496)^{{\cdot}5}$ etc., without the cumbersome method of ordinary logarithms, i.e., of having a negative characteristic, which must be made exactly divisible by the index. When quantities less than unity are being dealt with we work on the Reciprocal Scale No. 2, instead of the outer Multiplying Scale No. 1. as shown below in the solution of the above examples.

Ex. 22 : *Find the value of* $(0.310)^{2{\cdot}1}$

Because 0.310 is less than unity, and answer will be also, set 0.310 on Reciprocal Scale and read its log. on log. scale = 0.5085 (approx).

Multiply by 2.1 and take *wholly* negative since answer will be less than unity.

Then log. of answer $= -(2.1 \times 0.5085) = -1.0678$ (using Scale Nos. 1 or 5 for this multiplication, preferably the latter).

Set decimal part (.0678) on log. scale, and read answer on Reciprocal Scale which $= 0.0856$.

Ex. 23 : *Find the value of* $(0.496)^{0 \cdot 5}$

Because 0.496 is less than unity, and answer will be also, work on Reciprocal Scale and Log. Scale.

Set 0.496 on Reciprocal Scale, and read its log. (0.305) on Log. Scale No. 4.

Multiply by 0.5 and take *wholly* negative since answer will be less than unity.

Then log. of answer $= -(0.5 \times 0.305) = -0.1525$. Set 0.1525 on Log. Scale, and read answer on Reciprocal Scale $= 0.704$.

Exercises in Squares and Square Roots.—Scales No. 1 and No. 3.

The numbers on Scale No. 1 are the squares of the numbers on Scale No. 3, which extends round the inner and then the outer circumference of a common circle.

Conversely the numbers on Scale No. 3 are the square roots of the numbers on Scale No. 1.

The learner is advised to read the two scales, together with the aid of the cursor and compare numbers whose squares and square roots can be easily compared mentally so as to familiarise himself with the scales, e.g., 2 and 4, 3 and 9, 5 and 25, 8 and 64, etc., and afterwards try larger numbers.

The following table shows how the values of squares and square roots can be approximated mentally and so located on the scales of the Calculator :—

			Hence for any Number between	The Square Root is between
10^2	=	100	1 and 100	1 and 10
20^2	=	400	100 ,, 400	10 ,, 20
30^2	=	900	400 ,, 900	20 ,, 30
40^2	=	1600	900 ,, 1600	30 ,, 40
50^2	=	2500	1600 ,, 2500	40 ,, 50
60^2	=	3600	2500 ,, 3600	50 ,, 60
Etc.			Etc.	Etc.

If the number of digits in a number is odd the square root is read on the inner circle of Scale No. 3. If even it is read on the outer circle of that scale.

If a number is less than unity the square is less than the number and the square root greater, e.g.,

$(\frac{1}{2})^2 = \frac{1}{4}$ $(0 \cdot 8)^2 = 0 \cdot 64$ $\sqrt{\frac{1}{100}} = \frac{1}{10}$ $\sqrt{0 \cdot 49} = 0 \cdot 7$

Ex. 24 : *Find the square of* 0·7462.

Set cursor to 7462 on Scale 3.

Read Scale 1 under cursor.

We estimate value to be 557, and as the given number is less than unity, its square will be less and we write the answer, 0·557.

The correct value is 0·5568, but the reading on the scale is a close approximation.

Ex. 25 : *Find the square of* 27·52.

Set cursor to 2752 on Scale 3.

By mental estimation as per table the square is between 400 and 900.

Reading Scale 1 we make it 757·5.

Actually it is 757·3, a close approximation.

Ex. 26 : *Find the square root of* 1728.

The number lies between 1600 and 2500, therefore the root lies between 40 and 50.

Set cursor to 1728 on Scale 1.

It is between 17 and 18, which here stand for 1700 and 1800 ; the exact point is just short of the third graduation line after 17.

Read answer on outer circle of Scale No. 3 between 41 and 42 where each graduation counts 2.

It is just short of the third, which would be 41·6; we call it 41·58.

This is correct to four figures and is a good illustration of the extreme accuracy of the scale.

Ex. 27 : *Find the square root of* 0·00378.

As the number is less than unity the root will be larger than the number.

Set cursor over 378 on Scale No. 1. This is the graduation line preceding 380.

Read on Scale No. 3, outer circle, the result between 61 and 62 ; we estimate it as 6148 and the answer, therefore, as ·06148, correct to five figures.

If in Ex. 26 and 27 above we had been asked to give the reciprocals of $27 \cdot 52^2$ and $0 \cdot 7482^2$, i.e. the value of $\frac{1}{27 \cdot 52^2}$ and of $\frac{1}{0 \cdot 7482^2}$ they could have been read without further setting by simply reading the answers on Scale No. 2 (The Reciprocal Scale) instead of Scale No. 1 and shows how convenient this scale occasionally may be.

In Ex. 26 and 27, however, the values $\frac{1}{\sqrt{1728}}$ and $\frac{1}{\sqrt{0 \cdot 00378}}$ could not be given until the square roots, viz., 41·58 and 0·06148 had been first transferred to Scale No. 1 as reciprocal values can only be read from Scale No. 1.

Cubes and Cube Roots—Scales Nos. 1, 3, 5.

The third power or cube of a number can be easily obtained by multiplying itself three times $x \times x \times x = x^3$ or by reading x^2 on Scale No. 1 from Scale No. 3 and multiplying that power again thus ($x^2 \times x = x^3$).

The first method is the simpler and probably the better, as the same scale is used each time.

Cube Roots may be evaluated in a simple way at one setting by reading Scales Nos. 1, 3, and 5 in conjunction, from the fact that the cube root of any number is the same as the sixth root of the square of that number thus $\sqrt[3]{x} = \sqrt[6]{x^2}$

Now Scale No. 5 (The Long Scale) extends over six circles and gives the 6th roots of the numbers on Scale No. 1, which are the squares of numbers on Scale No. 3

The particular circle of Scale No. 5 on which a cube root is located can be determined mentally by a consideration of the following :—

	Hence for any Number between	The Cube Root is between
$10^3 = 1000$	1 and 1000	1 and 10
$20^3 = 8000$	1000 „ 8000	10 „ 20
$30^3 = 27000$	8000 „ 27000	20 „ 30
$40^3 = 64000$	27000 „ 64000	30 „ 40
$50^3 = 125000$	64000 „ 125000	40 „ 50

It is easy to say mentally the cube of 2, 3, 4, 5, and add three cyphers and so fix the blocks of numbers to which the cube root of any number between 1 and 50 belongs.

Ex. 28 : *Find the cube root of* 34,680 ($\sqrt[3]{34680}$).

Set cursor over 34680 on outer circumference of Scale No. 3. It is situated just a little less than three-quarters of the distance between the 3rd and 4th graduation following 34.

On Scale No. 1 the cursor lies over the square of the given number, but without paying any attention to the square, seek for the cube root on Scale No. 5, between 30 and 40, because the given number is situated between 27,000 and 64,000. A rapid survey shows that the cube root is under the cursor on the 4th circle reckoning from the centre and between 32 and 33. At this part the scale is graduated in twentieths and we make the reading, i.e., the answer, 32·61, which is as close an approximation as can be given in four figures.

Ex. 29: *Find the cube root of* $\frac{1}{6844}$ *i.e.* $\sqrt[3]{\frac{1}{6844}}$

First find $\sqrt[3]{6844}$ which as the number is between 1,000 and 8,000 must lie between 10 and 20, and proves to be actually 18·98. So what is required is the value of $\frac{1}{18{\cdot}98}$

Set cursor over 18·98 on Scale No. 1.

Read on No. 2 (The Reciprocal Scale) the answer under the cursor. This reading is 527.

From inspection of the fraction $\frac{1}{18{\cdot}98}$ we see its value is round about $\frac{1}{20}$ and, therefore, write down:

$$\sqrt[3]{\frac{1}{6844}} = 0{\cdot}05270.$$

Cube roots or any other roots or powers, whole or fractional, may be obtained by means of logarithms and sometimes this method is to be preferred.

LOGARITHMS.

The value of logarithms can be read definitely to three places of decimals, and sometimes four, by the use of scales 1 and 4 used in conjunction, but with the method to be described below, four and five figures can be obtained with ease and certainty.

Limitations of space prevent any lengthy exposition of the theory of logs, but the following notes are useful.

The logarithm of a number is composed of two parts, the Characteristic and the Mantissa.

The **characteristic** is the part of the logarithm to the left of the decimal point, and **may be positive or negative.**

If the number is greater than unity the characteristic is positive and one less in value than the number of figures to the left of the decimal point in the number.

If the number is less than unity the characteristic is negative and one greater than the number of cyphers to the right of the decimal point in the number. The indication of negative value is shown by a minus sign over the top of the characteristic.

The **Mantissa** is the part of the logarithm to the right of the decimal point and is **always positive,** and for the same figures always the same wherever the decimal point may be.

These features of logarithms are shown in the following examples :—

Log of	278 is 2·444.		Log of	0·278	is $\bar{1}$·444.
„	27·8 „ 1·444.		„	0·0278	„ $\bar{2}$·444.
„	2·78 „ 0·444.		„	0·00278	„ $\bar{3}$·444.

[A fuller description of logs., with tables of logs, and anti-logs., will be found in " Fowler's Machinists Pocket Book," Scientific Publishing Co., Manchester, 4/9 net. post, free.]

Ex. 30 : *Find logarithm of* 2675.

Using Scales Nos. 1 and 4.

Set cursor over 2675 on Scale No. 1. Read Mantissa of log., viz., 427 on scale 4. As there are fou figures in the number all to the left of the decimal point, the characteristic of the log. is positive and its value is 3.

The complete log is 3.427.

Ex. 31 : *Find logarithm of* 50·75.

Set cursor over 5075 on Scale No. 1 (it lies between the first and second graduation line after 5).

Read Mantissa of log. on Scale 4, viz., 7055.

The characteristic of the log. (as there are two figures to left of decimal point) is 1.

The complete log. is 1·7055.

Ex. 32 : *Find logarithm of* 0·024076.

Set cursor over 24076 on Scale No. 1. This is about one-third of the way between 24 (which represents 240) and the first graduation after it, which represents 2402.

Read mantissa of log. on Scale No. 4.

We make the reading 3815.

As the number is less than unity the characteristic is negative and as there is a cypher to the right of the decimal point its value is 2.

Therefore the logarithm of 0·024076 = $\bar{2}$·3815.

To find a number corresponding to a given logarithm·

Set the decimal portion of the logarithm on the Scale No. 4 under the cursor and read on Scsle No. 1 the corresponding number.

The index of the logarithm increased by 1 will be the number of integers in the given number when it is a whole number or the index dimished by 1 will be the number of prefixed cyphers when the number is a decimal fraction, and the index consequently negative.

Ex. 33 : *Thus given* 2.1880 *as the logarithm of a number.*

To find this number set 1880 (the decimal portion of the log.) on Scale No. 4 under cursor.

Read under cursor on Scale No. 1, 1542.

As 2 is the characteristic of the number there will be three whole figures to the left of the decimal point in the answer, which is therefore 154·2.

Ex. 34 : *Find the number which has* $\bar{4}$·5250 *as its logarithm.*

Set 5250 (the decimal portion of the log.) on Scale No. 4 under cursor.

Read under cursor on Scale No. 1, 335.

As the index is $\bar{4}$, diminishing this by 1 in an arithmetical sense gives 3 as a remainder, but increasing $\bar{4}$ by 1 in an algebraical sense gives $\bar{3}$, and 3 is just the number of cyphers after the decimal point of the number required, which thus becomes ·000335.

In the algebraical sense therefore the index is in every case increased by 1 to give the number of integers, or prefixed cyphers.

As explained above by the use of a method evolved some years ago by one of our calculator users — the late Mr Harold, Palmer M.P.S. of Cheltenham—the length of the log. scale may theoretically be increased to six times its normal length, which in the "Magnum" instrument is approximately 10·5 inches. We are thus favoured with a log. scale theoretically 63 inches in length, with a corresponding further increase of accuracy of reading,and both logs (mantissa portion), and antilogs, are read from the long scale.

A few examples showing the method of working are given below :—

Ex. 35: Find the log. of π (=3·1416).

(a) Place the cursor over π on the long scale. π appears on the 3rd circle from the centre. Then 3 less 1 = 2.

(b) Read the value on the logarithm Scale No. 4 under cursor as 983.

(c) Write down the result as 2·983.

(d) Now divide 2·983 by 6 = ·4971.
The log. of 3·1416 is therefore ·4971.

Ex. 36 : *Find the log. of* 437·5.

(a) Set cursor to 4375 on long scale. It appears on the 4th circle from centre. Then 4 less 1 = 3.

(b) Read the vslue on log. scale No. 4 under cursor as 846.

(c) Combining the two values as before = 3·846.

(d) Divide 3·846 by 6 = ·6410. This is the mantissa.
The log of 437·5 is therefore 2·6410.

It is surprising how quickly the process can be mentally effected, the dividing by 6, and the memorising by the subtraction of 1 from the circle on which the number whose log. has to be found is placed.

Ex. 37 : *Find the log. of* 1455.

(a) Place the cursor over **1455** on the long scale. This number appears on the 1st circle. Then 1 less 1 = 0.

(b) Value on the log. scale No. 4 under cursor = 978.

(c) Combining the two values as before = 0·978.

(d) Divide 0·978 by 6 = ·163. This is the mantissa.

The log. of 1455 is therefore 3·163.

It is of course, possible by reversing the process to find the antilog. of a number.

Thus find the antilog. of 2·6410.

(a) Multiply the **Mantissa** by 6, i.e., .6410 × 6 = 3·8460.

(b) Turn cursor to 846 on log. scale (No. 4).

(c) The answer is found on the 3 + 1 = the 4th circle under the cursor = 4375.

The Characteristic is 2 and 2 + 1 = 3. The answer therefore contains 3 digits and is 437.5.

Finding Nth Powers and Nth Roots of Numbers—with logarithms (whether N be a whole number or a fraction).

Let A be a number and suppose $x = A^n$

Where n may be a whole number or a fraction.

Then log. x = n log. A.

Ex. 38 : *Find 5th root of* 51·53 (*i.e. Find* $51{\cdot}53^{\frac{1}{5}}$)

Here n = 1·5th and A = 51·53.

Set cursor over 51·53 on Scale No. 1.

This is between the 3rd and 4th graduations after 5.

Read mantissa of log. on Scale No. 4, viz., 713.

The number is more than unity, therefore the log. is positive. There are two figures to left of decimal point, therefore the value of the characteristic is 1.

Therefore the log. of 51·53 = 1·713.

One fifth of log. of 51·53 = 0·3426.

Set cursor over 3426 on Scale No. 4.

Read 5th root of 51·53 on log. Scale No. 1, viz., 2·2.

Hyperbolic Logarithms.—These which are to the base e = 2·71828 are much used in calculations relating to the expansion of gases. They can be easily derived by multiplying the common logarithm (i.e., the log. to the base 10) by 2·30258.

The exact position of this multiplier denoted by $\log_e 10$ is indicated both on Scale No. 1 and Scale No.

5, but for purpose of finding common logarithms Scale No. 1 must be used.

Ex. 39 : ***Find hyperbolic log. of*** **14·35.**

First find common log. of 14·35.

Set cursor over 1435 on Scale No. 1.

Read mantissa of common log., viz., 1575 on Scale 4. As there are two figures to left of the decimal point in the number and the number is greater than unity, the characteristic is 1 and positive.

Therefore the log. of 14·35 is 1·1575.

Now multiply 1·575 by $\log_e 10$.

Set $\log_e 10$ on Scale No. 1 under datum.

Turn cursor to unity and then turn dial till 11575 comes under cursor.

Read answer 266 = hyperbolic log. under datum.

To convert hyperbolic logs. to common logs. multiply by 0·43429 indicated as log. 10e in the gauge points.

Note—If greater accuracy is required the method of finding log. values from the Long Scale as given previously would, of course, be used.

EXTRACTING ROOTS OF LOGARITHMS

Roots of numbers as for example $\sqrt[2]{162}$ or $\sqrt[3]{9176}$ can easily be extracted by means of logarithms. The small(2) and (3) of the root sign is called the index of the root, and any sort of number may be found by dividing its logarithm by this index. The quotient obtained is then the logarithm of the root.

Ex. 40 : *Find* $\sqrt[3]{694}$.

Set cursor over 694 on Scale No. 1, and read mantissa of log. on Scale No. 3, vis., ·8414. As there are 3 whole numbers in 694 the characteristic will be 2 and the log. of 694 will therefore be 2·8414.

2·8414 ÷ 3 = 0·9471

Hence log. $\sqrt[3]{694}$ = 0.9471

Set 9471 on Scale No. 4 under cursor and read on Scale No. 1 885. By inspection we should say that $^3\times 694$ is therefore 8·85.

Ex. 41 : *Find* $\sqrt[5]{0{\cdot}82}$.

Set cursor over 82 on Scale No. 1. Read on Scale No. 4 mantissa of log. = ·9138.

Therefore log. 0·82 = $\bar{1}{\cdot}9138$.

In this case it is not possible to divide directly by 5 because there is a negative characteristic, and a positive mantissa. The artifice of adding numerically as many **negative** units, or parts of units, to the characteristic as is necessary to make it evenly contain the index of the root is adopted. The same number of **positive** units or parts of units is then added to the mantissa, and each is then separately divided by the index.

Thus $\bar{1} + \bar{4} = \bar{5}$ and $\bar{5} \div 5 = \bar{1}$.
$\cdot 9138 + 4 = 4\cdot 9138$ and $4\cdot 9138 \div 5 = \cdot 9827$
$\bar{1} + \cdot 9827 = \bar{1}\cdot 9827 = \log.. \sqrt[5]{0\cdot 82}$
Therefore $\sqrt[5]{0.82} = 0.9610$
obtained by setting ·9826 on Scale No. 4 under cursor and reading this value 0·9610 on Scale No. 1.

Ex. 42 : *Find* $\sqrt[2\cdot 4]{0\cdot 6}$

Set cursor over 6 on Scale No. 1 and read on Scale No. 4 mantissa of log. $= \overline{778}$.

Characteristic will be $\bar{1}$. Therefore log. $0\cdot 6 = \bar{1}\cdot 778$. If we add (—1·4) to the characteristic of this log. it will be evenly dividible by the index of the root (vis., 2·4.) Hence $\bar{1} + (-1\cdot 4) = -2\cdot 4$ and $-2\cdot 4 \div 2\cdot 4 = \bar{1}$. Now add $+1\cdot 4$ to the mantissa portion of log. and divide this by the index of the root.

Thus $\cdot 778 + 1\cdot 4 = 2\cdot 178$ and $2\cdot 178 \div 2\cdot 4 = \cdot 9075$.

Adding these together we get $\bar{1}\cdot 9075$.

Therefore log. $\sqrt[2\cdot 4]{0\cdot 6} = \bar{1}\cdot 9075$.

Now set 9075 on Log. Scale No. 4, and read $\sqrt[2\cdot 4]{0\cdot 6}$ on Scale No. 1 $= 0\cdot 808$.

Extracting Square and 4th Roots by means of Scales Nos. 1 and 2 used in conjunction.

Ex. 43 : *Find the square root of* 1849.

Set 1849 on Scale No. 1 under datum.

Set cursor to ·1.

Turn dial anti-clockwise until the same number comes simultaneously under the datum on Scale No. 1, and the cursor on Scale No. 2. This number is 43 the square root of 1849. Opposite 43 on either Scale No. 1 or Scale No. 2 will be found the value of $\dfrac{1}{\sqrt{1849}}$ vis.: 0·02325.

It will be observed that two values of square root may be obtained in this way. For instance in above example we can get either 43 coming on Scales Nos. 1 and 2, when the unity line on the dial comes opposite the mid-point between datum and cursor, or we could get 136 when unity line falls midway between the datum and cursor.

The other value, for example the 136 givan above, is the square root of the original number (1849) multiplied by the square root of 10.

Thus $136 = \sqrt{1849} \times \sqrt{10}$

Ex. 44 : *Find the 4th root of* 1849.

Proceed as in Example 36 above to find the square root (43) and then obtain the square root of 43 in a similar manner.

Set 43 on Scale No. 1 under datum.

Set cursor to 1.

Turn dial anti-clockwise until the same number comes simultaneously under the datum on Scale No. 1 and the cursor on Scale 2. Thus 6·56 is the 4th root of 1849.

Obtaining Powers of Numbers by Logarithms.

To raise a number to a given power multiply the logarithm of the number by the power index; the product is then the logarithm of the result.

Ex. 45 : *Find the value of* $27\cdot 5^3$

Here the log. of the result will be 3 times the log. of 27.5.

Set cursor over 27·5 on Scale No. 1.

Read log. on Scale No. 4 (·4393 and add characteristic (1) = 1·4393

$3 \times 1\cdot 4393 = 4\cdot 3179$

Set cursor over ·3179 on Log. Scale No. 4.

Read 2079 on Scale No. 1 under cursor.

Since the characteristic is 4 we must add another cypher to this result to make 5 figures to the left of the decimal place. The answer is therefore 20790

The correct arithmetical answer is 20796·875.

Ex. 46 : *Find the value of* $16^{1\cdot 33}$

Set cursor over 16 on Scale No. 1 and read log. on Scale No. 4 adding characteristic (1) to same. This equals 1·204.

Multiply 1·204 by 1·33 on Scale No. 1.

Set 1·204 to datum.

Set cursor to unity.

Set dial till 1·33 comes under cursor.

Read $1\cdot 204 \times 1\cdot 33 = 1\cdot 60$ on Scale No. 1.

Set 60 on log. Scale No. 4 under cursor.

Read 3981 on Scale No. 1 under cursor.

Since characteristic will have 2 figures to left of decimal point the answer is 39·81.

TRIGONOMETRICAL VALUES.

Sines, Tangents, etc.—The values of sines, tangents, etc., are read from the scale of angles No. 6 and No. 7 by means of the cursor.

Read Natural Sin or Natural Tan on Scale No. 1.

Read Log. Sin or Log. Tan on Scale No. 4.

Cosine, Cotangent, Secant and Cosecant are deduced from Sine and Tangent through the following relationships.

For any given angle A :—

$$\text{Cos A} = \text{Sin}(90—A); \qquad \text{Cot A} = \frac{1}{\text{Tan A}}$$

$$\text{Sec A} = \frac{1}{\text{Cos A}} \qquad \text{Cosec A} = \frac{1}{\text{Sine A}}$$

The Scale of Sines, No. 6, extends twice round the circumference of a circle. The inner circle gives angles between 35 mins. and 5 degs. 45 mins. and the value of the sine increases from 0·01 to 0·10. The outer circle gives angles between 5 degs. 45 mins. and 90 degs., and the value of the Sine increases from 0·10 to 1·0.

Ex. 47 : *Find value of Natural Sine of* 4° 40′.

Set cursor over 4° 40′ on Scale No. 6.

Read Natural sine 0·0813 on Scale No. 1.

N.B.—The number on the scale is 813, but as the sines of all angles on the inner circle of Scale No. 6 are between 0·01 and 0·1 we write down the value 0·0813.

Ex. 48 : *Find value of Natural Sine of* 20° 30′.

Set cursor over 20° 30′ on Scale No. 6.

Read value of Natural Sine 0·3502 on Scale No. 1.

N.B.—The angle being on the outer circle of Scale No. 6, and the angle exceeding 5° 45′, the value of the sine is between 0·1 and 1·0.

Between 20° and 25° the scale is graduated at intervals of 20′ so that 20° 30′ falls midway in the second interval following 20°.

In reading the values of log. sines of angles the characteristic of the logs. for all angles between 35 mins. and 5 degs. 45 mins. is 8, and for all angles between 5 degs. 45 mins. and 90 degs. is 9.

The mantissa only of the log. is read on Scale No. 5.

Ex. 49 : *Find value of Log. Sine of* 27° 20′.

Set cursor over 27° 20′ on Scale No. 6.

Read mantissa of log. sine on Scale No. 4 = 662.

As the angle is on the outer circle of the Sine Scale the log. sine is 9·662.

Ex. 50 : *Find value of Log. Sine of* 4° 25′.

Set cursor over 4° 25′ on Scale No. 6.

Read mantissa of log. sine on Scale No. 4 = 8865.

As the angle is on the inner circle of the Sine Scale the complete log. sine is 8·8865.

Ex. 51 : *Find value of Natural tangent of* 31° 45′.

Set cursor over 31° 45′ on Scale No. 7.

Read value of Natural tan. on scale 1 = 0·6188.

Triangles : Useful Notes.—If A, B, C, is a triangle, the angles of which are A, B, C, and the sides opposite these angles are respectively a, b, c.

Then A + B + C = 180°.

$$\frac{a}{b} = \frac{\sin A}{\sin B} \qquad \frac{b}{c} = \frac{\sin B}{\sin C} \qquad \frac{c}{a} = \frac{\sin C}{\sin A}$$

$$a^2 = b^2 + c^2 - 2\,bc \cos A.$$

N.B.—Cos A is itself minus and the whole of the last factor becomes plus if A is greater than 90°.

If A is 90° the last factor disappears.

Sin (180—A) = sin A.

Cos (180—A) = —cos A.

Cotan A = tan (90—A).

$$\text{Area of a traingle} = \frac{a\,b \sin C}{2}$$

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\,\text{cotan}\frac{C}{2} \qquad \text{cotan}\frac{C}{2} = \tan\frac{A+B}{2}$$

These last two formulae are much used for solving triangles when two sides and included angle are known.

If in a triangle A, B, C, the angle C is a right angle (90°) and the sides opposite the angles are respectively a, b, c (letters arranged clockwise).

$$\text{Sin A} = \frac{a}{c} \qquad \sin B = \frac{b}{c} \qquad \cos A = \frac{b}{c} \qquad \cos B = \frac{a}{c}$$

$$\tan A = \frac{a}{b} \qquad \text{cotan A} = \frac{b}{a} \qquad \text{secant A} = \frac{a}{b}$$

$$\text{cosecant A} = \frac{c}{a}$$

If C is not a right angle the sine, cosine, etc., still have same values, but a and b are not now the sides of the actual triangle, but of an imaginary triangle with B C perpendicular to C A.

A circle is divided into 360 degrees. Each degree is divided into 60′ (minutes) ; each minute is divided into 60″ (seconds), but seconds are rarely considered.

MENSUATION OF CIRCLES

Ex. 52 : *Find the area of a circle* 3½ *inches diameter*

Area = $d^2 \times \pi/4$ = 3·5 × 3·5 × ·7854.

Set 35 on Scale No. 1 under datum.

Set cursor to 35 on Scale No. 2.

Turn dial till π/4 (gauge point on outer cirlce) comes under cursor.

Read area 9·62 square inches on Scale No. 1 under datum.

Ex. 53 : *Find circumference of circle* 9·3 *inches diam.*

Set 93 on Scale No. 1 under datum.

Set cursur to 1.

Turn dial till π (gauge point on outer circle) comes under cursor.

Read circumference 29·2 under datum on Scale No. 1.

This and following examples may, of course, be worked out on the Long Scale (No. 5) in a similar manner; the value of π (3·1416) as well as the diameter being taken on the Long Scale. This will give a close approximation, vis., 29·22 nearly.

Ex. 54 : *Find the diameter of a circle whose area is 227 square inches.*

Dia. = $\sqrt{}$ Area × C. C = 1·12838 and is marked as a gauge point on outer circle.

Set 227 on Scale No. 1 under datum.

Set cursor to 1.

Turn dial till same number (15·07) comes under datum on Scale No. 1 and under cursor on Scale No. 2. This is the square root of 227.

Turn cursor to 1.

Turn dial till " C " comes under cursor.

Read answer 17 under datum on Scale No. 1.

PROBLEMS IN PERCENTAGES

In speaking of percentages confusion often arises through inattention to the basis on which it is measured. If A's salary is £75 and B's £50 it would be true to say A's salary was 50 per cent. greater than B's, and equally true to say that B's salary was 33 per cent. less than A's. The fact is only expressed in two different ways. There can be no misapprehension in any case if the quantity representing the 100 is made clear. Set the question as a problem in fractions thus :—

Ex. 47 : In an examination 27 scholars pass 1st class 35 2nd class, and 63 3rd class. Express the various numbers as percentages of the whole.

Here 27 + 35 + 63 = 125 and this total must be regarded as a 100 base which has to be divided into three similar proportions. Therefore if *x*, *y*, *z* are the three percentages, we have the following relationships :

$$\frac{27}{125} = \frac{x}{100} \text{ and } x \; \frac{100 \times 27}{125} = 21 \cdot 6 \text{per cent.}$$

$$\frac{35}{125} = \frac{y}{100} \text{ and } y = \frac{100 \times 35}{125} = 28 \text{ per cent.}$$

$$\frac{63}{125} = \frac{z}{100} \text{ and } z = \frac{100 \times 63}{125} = 50 \cdot 4 \text{ per cent.}$$

For this class of question the instrument is very convenient. Set 1·0 on Scale No. 1 under datum and set cursor to 125 (i.e., 12·5). Rotate the dial until the several number 27, 35, 63, come under the cursor, and read the several percentages under the datum.

PROBLEMS IN PROPORTION

Set the question in simple fractional form as follows: Where A, B, and C are certain known quantities and *x* is the unknown quantity $\frac{A}{B} = \frac{C}{x}$

Each of these quantities may be in the numerator or the denominator as the operator finds it convenient in setting down their relationship but this must, of course be expressed correctly. Then by cross-multiplication we have :—

$$A \times x = B \times C \text{ and } x = \frac{B \times C}{A}$$

Ex. 55 : If 15 men do a task in 28 days, in how many days will 21 men do it, assuming they work at the same rate. Obviously more men will take less time in the ratio of 15 to 21 and if *x* is the number of days, then

$$\frac{x}{28} = \frac{15}{21} \text{ and } x = \frac{28 \times 15}{21} = 20 \text{ days.}$$

This and the following example are worked out on the Calculator as previously described under "Fractions."

Ex. 2 : If a task takes 18 men 36 days, how many men will be required to do it in 27 days ?

Obviously more men will be required in proportion to the increased speed at which the task must be done

$$\text{therefore } \frac{36}{27} = \frac{x}{18} \text{ and } x = \frac{36 \times 18}{27} = 24 \text{ men.}$$

DISCOUNT

Ex. 57 : What is the wholesale price of an article subject to a discount of 20 per cent., the retail price of which is 15/-.

Set unity line to datum.

Set cursor to 15.

Turn dial to 80 (20 backwards) representing 20 per cent.

Read 12/- under cursor.

Profit on Returns.—Supposing a merchant can produce an article at 6½d. per lb. and wishes to make a profit of 12½ per cent. **on Returns** the selling price can be determined as follows :—

Set unity line to datum.

Set cursor to 100—12½ = 87·5.

Then by setting any cost price under the cursor we can read the **Selling Price** under datum.

Thus $\frac{100 \times 6\cdot125}{87.5} = 7\text{d.}$

This can be worked out on either the Short or Long Scale.

Also while in this position we can place any Cost Price under the cursor and read the Selling Price (with 12½ per cent. profit) under the datum.

Thus cost		Selling price	
= 6¾d.	(6·75d)	= 7·719d.	(7¾d. approx.)
= 8$\frac{1}{16}$d.	(8·0625d.)	= 9·1875d.	(9$\frac{3}{16}$d.)
= 4$\frac{13}{16}$d.	(4·8125d.)	= 5·5d.	(5½d.)
= 24½d.	(24·5d.)	= 28d.	(2/4d.)

and so on.

It is recommended that the Long Scale be used for these problems as greater setting and reading accuracy can be obtained thereby. If any other percentage of profit is required the working will be the same, but the percentage must always be subtracted from 100 and the cursor placed to that figure.

Profit on Cost.—Assuming an article to cost 6½d. per lb. Find the selling price with the profit of 12½ per cent. on cost.

The working is as follows : 100 + 12½= 112½

Set 112.5 under datum on Long Scale.

Now by bringing any cost price on Long Scale under the **cursor** we get the selling price under the **datum.**

Thus cost		Selling price	
= 5d.		=	5·625d.
= 7¾d.	(7·75d.)	=	8·719d.
= 23¾d.	(23·75d.)	=	26·75d.

The principle can be adapted to many uses.

For instance, if a merchant has an offer of 9½d. per lb. and he can produce at say 8$\frac{11}{16}$d. per lb. his profits on *Returns* will be obtained by calculating as follows :—

(Use Long Scale.)

Set 9½d. under datum.

Turn cursor to 8$\frac{11}{16}$d., i.e., 8·6875.

Turn dial till 100 (unity comes under *Datum.*

Read 91·5 under *Cursor.*

The difference between datum and cursor will show the amount of profit on *Returns,* vis., 8½ per cent.

By turning 100 under *cursor* and reading 109.33 under the datum we read a profit of 9.33 per cent. *on cost.*

Again suppose the offer was for 9½d. per lb. and the merchant wishes to make 12½ per cent. profit *on: Returns* he will have to produce the goods at the following price.

Set 100 (unity) under datum.

Turn cursor to 100 — 12½ = 100 — 12.5 = 87.5.

Now by setting the offered price, vis. 9½d., under the datum the figure which comes under the cursor will be the price at which the goods must be produced.

The answer is 8·1875 = 8$\frac{3}{16}$d. = *Production Price.*

If an offer of 9½d. per lb. is made, and a profit of 12½ per cent. *on cost* is required the rule is :

Set 112½ (112·5) under datum.

Turn cursor to 100 (unity).

Set price offered under datum.

Read Production Price under cursor = 8·4375 = 8$\frac{7}{16}$d.

Hints on Arithmetical Calculations: Fixing Decimal Point.—A rough idea of the result of a calculation is often known beforehand, or if not, the position of the decimal point, where necessary, can be approximated by a rough survey of the fraction expressing the required calculation. There are rules, but they are more trouble to remember than they are worth. It is better for the operator to rely on first principles and rapid mental arithmetic.

For example, suppose the value of the following were required :—

$$\frac{6\cdot92 \times 746 \times 19\cdot2 \times 9}{2876 \times 92\cdot5}$$

we could reason mentally, and roughly, as follows :— 6·9 is practically 7, and 7 into 2,876 is roughly 400, 400 in 746 is roughly 2 ; 2 into 92·5 is roughly 45. This would be in the denominator, and for the numerator we should still have left 19·2 × 9, roughly 170. This divided by 45 would obviously give a value less than 10. In putting down the answer, therefore, we should write all figures after the first one to the right of the decimal point. A rough estimate like this occupies less time to make than to describe, and is safer than any cut-and-dried rule.

A METHOD FOR DETERMINING THE CIRCLE ON WHICH TO FIND THE ANSWER

A method for finding the circle on which to read the answer when using the "Long Scale" of a Fowler's Calculator, and which obviates having to previously work out the problem roughly on the "short" Scale, has been devised by Mr. D. Gordon Bagg, B.Sc., F.R.I.C., M.I.Chem. E.,and we give it here,with his kind permission.

Provided the complications mentioned are carefully observed, the method, which has been in use by him for many years, is infallible, and can be likened to the handling of log characteristics.

The " Long Scale " consists of six concentric circles which for the purposes of this method should be numbered consecutively from 1 to 6, commencing with the inner circle. Briefly, to find the circle on which the answer occurs, the numbers of the circles carrying the various numerators and denominators should be added together or subtracted according to whether the operation is multiplication or division. Thus, if the operation is a simple division of say 6 by 3, the number 6 on the 5th circle is set under the datum line, and the cursor is set to 3 on the 3rd circle. Subtracting the number of the denominator circle from the number of the numerator circle, the answer will be on the 5-3 or 2nd circle when the unity line is set below the cursor. If the operation is the multiplication of the same two numbers, 6 is set under the datum line, and the cursor is set to unity. The 3 is then set under the cursor, and the answer will be on circle 3 plus 5, i.e. circle 8. As there are only 6 circles, the 7th and 8th will be a return to the 1st and 2nd, so that the answer will appear on the 2nd circle. If the circle number of the denominator is greater than that of the numerator, 6 must be added to the latter to make a straight subtraction possible. Thus if 24 (3rd circle) is to be divided by 60 (5th circle), the answer (0·4) will be on the (3+6)—5, i.e. the 4th circle.

A complication occurs when the unity line on the scale appears between the red datum line and the cursor line, working in a clockwise direction from the datum line. The number appearing below the cursor must then be treated as though on a continuation of the corresponding circle below the datum line, i.e. the jump in circle number at the unity line must be ignored. For example, if 30 is to be divided by 25, the former number on the 3rd circle is set below the datum line and the cursor is set to the latter. 25 appears on the 3rd circle, but as in this case the unity line appears between the datum line and the cursor in a clockwise direction from the former, the jump from 2nd to 3rd circle at 21·54 must be ignored and the 25 must be assumed to appear on the 2nd circle. The answer (1·2) then appears on the 3-(3-1) or 1st circle.

Similarly, if the calculation is commenced by setting the unity line below the datum line, it must be treated as though it is set on the right of the datum line, i.e. the number of the circle to which the cursor is next set must be reduced by 1. For example, suppose several numbers are to be divided by 24. The unity line is set below the datum line and the cursor is set to 24, which appears on the 3rd circle. For the reason stated above, 24 is taken

as being on a continuation of the 2nd circle, and 2 is therefore subtracted from the numbers of the circles on which the various numerators appear. Thus if the numerator is 96 (6th circle) the answer appears on the 6-(1-3) or 4th circle. If the numerator is 72 (also 6th circle) the unity line appears between the datum line and cursor, and the answer will be on the (6-1)-(3-1) or 3rd circle.

The following calculation embodies the several points raised

$$\frac{75\times 36\times 12\times 64}{16\times 30\times 24\times 90\times 50}$$

and the operations are as follows :—

1. Set unity line below datum line.
2. Set cursor to 16 on 2nd circle. The circle number to be subtracted is 2-1, i.e. 1.
3. Set 75 (6th circle) below cursor. Subtract 1 (from operation 1) from 6. The answer so far is on circle 5.
4. Set cursor to 30 (3rd circle). Subtract 3 from the 5 resulting from the last operation. The answer so far is on circle 2.
5. Set 36 (4th circle) below cursor. Unity line appears between datum line and cursor, so add (4-1). i.e. 3, to the 2 from the last operation. The answer now appears on circle 5.
6. Set cursor to 24 (3rd circle). Unity line still appears between datum and cursor, so subtract (3-1), i.e. 2, from the 5 from last operation. Answer is now on circle 3.
7. Set 12 (1st circle) below cursor. Unity line still appears between datum and cursor, so add (1-1) i.e. nothing, to 3. Answer is still on circle 3.
8. Set cursor to 90 (6th circle). As 6 or multiples thereof can be ignored in the final circle number, no attempt need be made to subtract 6 from 3, and the answer is still on the 3rd circle.
9. Set 64 (5th circle) below cursor. Adding 5 to the last 3, the answer should now appear on the 8th circle. This exceeds 6 by 2, so that the answer now appears on the 2nd circle.
10. Set cursor to 50 (5th circle). Unity line again appears between datum line and cursor, so that 50 must be taken as on the 4th circle. 4 cannot be substracted directly from 2, and 6 must be added to the circle number resulting from operation 9. The answer is thus on the (2+6) — (5-1), i.e. the **4th circle.**

11. Set unity line below cursor as there is no further numerator. This does not affect the location of the answer (0·4), which is therefore on the 4th circle.

With a little practice, these additions and subtractions can be carried out mentally during the manipulation of the calculator, with the result that the answer can be found on the long scale without first carrying out the calculation on the short scale.

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