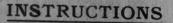
THE

FOWLER

"Jubilee Magnum"
EXTRA LONG SCALE

CALCULATOR

1898 - 1948



With numerous Examples
By
HAROLD FOWLER
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FOWLER'S (Calculators) LTD.

(Established 1898)

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Fowler's "JUBILEE MAGNUM"

Extra Long Scale Calculator

This instrument has been designed by the writer to mark the 50th anniversary of the founding of the firm by his father, the late William Henry Fowler, Wh.Sc., M.I.C.E., M.I.Mech.E., etc., and as a little tribute to the pioneering work he did in the development of circular calculators.

It is also intended to meet the wants of a great number of people whose primary requirements are for a device for solving problems of multiplication and division in a rapid and accurate manner. It is true to say that the bulk of engineering and scientific problems are of this kind, and the space occupied in our Standard "Magnum" Instrument by the Sine, Tangent, and Square Root Scales has therefore been utilised to increase the length of the "Long Scale," and with this increase of length to give a greater degree of subdivision.

The instrument is unique in several respects, for it gives in its small compass the advantage of a scale 79 inches in total length, on which calculations can be made accurately to four and sometimes five or six, significant figures. The long and finely, yet clearly divided logarithmic scale, enables log. values to be easily determined to four significant figures, and from these all calculations involving powers and roots of numbers can be made. The inclusion of a reciprocal scale not only enables decimal equivalents of fractions to be obtained, but this scale when used in conjunction with the outer primary scale, for multiplication and division, reduces by about half, the number of movements, and thus the time taken in calculations.

DESCRIPTION OF SCALES

The Instrument contains 4 separate scales arranged concentrically on a single dial so that they are all rotated together by a knurled nut at the top. A number of "gauge points" are also given, around the outer circle. These are certain multiplying factors which often occur, and to save time and trouble in setting them it is convenient to have their values so marked. It is hoped the selection given will be found useful.

All the scales have a common unity line which rotates with them, and there is also a common "Cursor" line which is rotated by a knurled nut at the side. A fixed Datum line is etched on the underneath surface of the

cover glass.

Commencing with the largest circle and proceeding

inwards to the smallest they are as follows:-

Scale No. 1.—The "Short-Scale," a single circle, a linches in circumference, graduated clockwise. This is the calculating scale for multiplication, division, etc., analogous to the ordinary slide rule. It is also used for direct reading and incorporating values given on other scales by aid of the cursor or datum line (the cursor by preference, as it is closer to the scales and climinates parallax).

Between the prime numbers 1 and 2 the scale is divided into 20 figured parts (11, 12, 13,) each decimally graduated and capable of further graduation with the cursor. Readings on this part can be made easily to four, and sometimes five significant figures.

Between the prime numbers, 6, 7, 8, 9, 10, each part is divided into 2, viz., 65, 75, up to 95, and each of these decimally graduated, so that on this, the finest part of Scale No. 1, readings can be made to three and

sometimes four significant figures.

Scale No. 2.—The "Reciprocal Scale," a single circle, exactly like No. 1, but graduated contra clockwise so that the readings on one are the reciprocal values of those radially opposite on the other. On this scale the values increase from right to left instead of from left to right as in all the other scales. It should be noted that any value on Scale No. 4 (the Long Scale) must be expressed on Scale No. 1 before its reciprocal value can be read on Scale No. 2.

Scale No. 3.—A Scale of Logarithms uniformly graduated from 0.002 onwards, giving 500 divisions in the circumference, and as each of these divisions can be split with the Cursor line, values can be read accurately to three significant figures at least, and four and often five figures may be obtained by the use of the long scale as will be explained later. The scale is used in conjunction with Scale No. 1 as follows:—If the logarithm of a number is required the cursor line is set over the number on Scale No. 1, and the mantissa of the log, of the number is read on Scale No. 3 under the cursor. The characteristic of the log, is determined as explained in the chapter on Logarithms.

Conversely for a given common logarithm read on Scale No. 3 the corresponding number is read on

Scale No. 1.

Scale 1 . 4 .- The "Long Scale." A scale extending round to circles, beginning at the smallest and contiming round the successive circumferences until it completes the tenth, with a tot I length of 79 inches The most v luable feature of this scale is its great length, which permits of multiplication and division with a degree of accuracy beyond the possibility of any straight slide rule. The scale is used just like No. 1 in setting factors, but requires a little mental consideration like any other logarithmic scale to determine the result, and hence the circle on which it is to be read. This is more fully discussed in describing the practical use of the scale. Its great length enables it to be divided into 270 figured parts, 1, 102, 104, 106, ... up to 1,000, and each of these to be graduated decimally, which makes reading very simple.

Graduations are made at intervals of -002 from unity to the primary number 4, and from 4 onwards to the

end of the scale at intervals of .005.

Three significant figures of a result can be written without hesitation even in the most finely graduated part between 99 and 100, while further division can be made with the cursor, and over a great part of the scale results can be read to four and sometimes five significant figures. The superiority of such a scale over that of the 10 inch slide rule will be manifest to those familiar with that instrument which only permits 2 graduations between 99 and 100 as compared with the 20 graduations of the Long-Scale of the "Jubilee Magnum."

Multiplication and Division.—The great bulk of calculating work consists of multiplication and division. These operations are in essence addition and subtractine even in ordinary arithmetic and with the Calculator are reduced to these elementary principles. The working out of a compound fraction, for instance, containing,

several fractions in the numerator and in the denominator resolves itself into rotating the added numerator factors in one direction, and the subtracted denominator

factors in another direction.

Just as there are several ways of working out a compound fraction sum by arithmetic, there are several ways of doing it with a calculator. The numerator factors may be all multiplied together and divided by the total product of the denominator factors, or the factors of the numerator and denominator may be dealt with in pairs, one after the other. Sometimes one method is better than another. The user will discover best methods and short cuts for himself as he acquires proficiency. His first step is to master the principles of operation by studying a few practical examples worked out and described in detail.

Fx. 1: Find the product of the factors a, b, c, d.

Either the Short Scale, No. 1, or the Long Scale, No. 4, may be used. For this illustration Scale No. 1 will be used.

Set factor a under datum.

Set cursor to one.

Set dial till factor b comes under cursor.

Read product $a \times b$ under datum.

Next set cursor again to one.

Set dial till factor c comes under cursor. Read products $a \times b \times c$ under datum.

Read products $a \times b \times a$

Set dial till factor d comes under cursor.

Read product $a \times b \times c \times d$ under datum.

The process after setting the first factor a under the datum is a succession of settings of cursor and of factors on the scale, and of finally reading the product under the datum. The whole operation begins at the datum and ends there. It consists in sum of turning the several factors past a fixed point, and reading the total of angular movements at the end. It matters not whether the angular movements of the dial and cursor are made clockwise or contra-clockwise for the individual settings. So long as the sequence is in the order stated the reading of the final result is the same

If there are decimal points in the factors the position of the point in the final product is to be decided by inspection and mental consideration as with all logarithmic work. Actual examples of this will be given in the course of the exercises.

If Scale No. 4 (Long Scale) is used instead of Scale No. 1 (Short Scale) the succession of operations is precisely the same, but the setting calls for a little more

care as the factors are spread over a scale extending round ten circles, and the answer may also be on any one. The particular circle must be determined by a mental consideration of the factors or it may be determined by first roughly and rapidly working out the problem on the short scale, no special care bei g taken in setting the factors to either datum or cursor. An approximate answer will then result, but one sufficiently accurate to give the location of it, when worked out on the long scale, with care taken in all settings. Such a quick trial on the short scale can be made in less time than it takes to describe, but it eliminates any reading from a wrong circle when the long scale is used. When doing tabular work or working out a series of scientific results the location of the first often serves as a guide to the location of the following ones, the Long Scale then can often be read just as easily as the Short Scale, without preliminary trial.

The great advantage of the Long Scale is its fine and uniform graduation, and this, with its great length, makes for easy setting and reading, coupled with extreme accuracy.

To get accustomed to reading Scales Nos. 1 and 4 and their graduations the learner will find it good practice to work through a multiplication table such as twice one are two, twice two are four, etc., thus:

Set 2 on Scale No. 1 under datum and set cursor to

Then turn, in succession, all the figured graduations past the cursor, noting that the procession of values which pass the datum are twice those which pass the cursor.

Thus $2 \times 11 = 22$; $2 \times 12 = 24$; etc.

Do the same for 3, or other simple number and proceed

to such multipliers as 3.1, etc.

This kind of exercise teaches the learner to read accurately parts of the scale that are not figured or where the graduations require to be split in reading and each counted as 2 if there are 5 graduations, or each counted as 5 if there are 2 graduations.

Division.—This is in essence subtraction, and the reverse of multiplication, which is addition. Its performance with the Calculator is best acquired by practising with simple fractions till the routine of operations becomes mechanical.

Assume the division is of a simple fraction form

mi.e., with a single numerator and a single denominator, and also that Scale No. 1 is being used, and the learner is advised to get accustomed to the Scales Nos. 1 and 2

before using Scale No. 4. Set a under datum. Set cursor to m Set one to cursor.

Read value of $\frac{a}{m}$ under datum.

Next consider the fraction $\frac{a \times b}{m}$ with two factors in the numerator and one in the denominator

Set dial till a comes under datum

Set cursor to m

Set dial till h is under cursor

Read answer under datum

Now consider a fraction with several factors in the numerator as well as in the denominator. It makes no difference to the answer whether all the top factors are multiplied together, and then divided by all the bottom factors multiplied together; or, whether the top and bottom factors are worked in pairs as single fractions. and finally united in a group.

Obviously also the insertion of the factor 1 in the numerator or denominator can make no difference to the result. Learners will, however, find it easier, at first, to work such a compound fraction by taking the factors alternately from numerator and denominator, and to enable them to do this in a routine manner the factor I should be inserted in the fraction as often as may be necessary to make the numerator contain one more factor than the denominator. Any compound fraction can then be worked as follows :-

 $\frac{a \times b \times c}{m}$ work as $\frac{a \times b \times c}{m \times 1}$

Set factor a under datum.

Set cursor to m.

Set factor b to cursor.

Set cursor to 1.

Set factor c to cursor.

Read answer under datum. $\frac{a \times b}{m \times n}$ work as $\frac{a \times b \times 1}{m \times n}$

setting the factors as in the previous example.

 $\frac{a \times b}{m \times n \times p}$ work as $\frac{a \times b \times 1 \times 1}{m \times n \times p}$

and set the factors in a similar manner.

It will be observed in all these three examples:-The factors are taken alternately from the numerator and the denominator beginning with the numerator.

The dial is always turned for multipliers.

The cursor is always turned for divisors The datum is used only to set first factor and to read final result.

PRACTICAL WORKED-OUT EXAMPLES.

Multiplication of Two Factors on Short Scale No. 1. Ex. 1: Multiply 12.8 by 5.62.

Set 12.8 on Scale No. 1 under datum

This is the 8th graduation past the 12

Set cursor to Unity line

Set dial till 5.62 on Scale No. 1 comes under cursor This is between the 55 and 6 mark; the exact point being 6 divisions past the 55 to make the 56 and the 9 of 569

Read answer just under 72 on Scale No. 1 under datum

We should estimate this as 71.9

The exact answer by ordinary multiplication is 71.936 Multiplication of the same Two Factors on the Long Scale No. 4.

Set 12.8 (marked 128) on the 2nd circle from the centre under datum

Set cursor to Unity line.

Set dial till 5.62 on the 8th circle from the centre

comes under cursor This is the 4th graduation past the 56 mark, i.e., the

2nd long line. Read answer just before the 7195 mark on Circle

No. 9 from centre We should estimate this as a little under 7194, and

perhaps give it a value of 71.935, which would be nearly exact. Ex. 2: Multiply .0347 by 2.8 on the Short Scale No. 1.

Set 347 on Scale No. 1 under datum

This lies between the 34 and 36 marks: the exact point being half-way between the 3rd and 4th graduations past the 34.

· Set the cursor to Unity line.

Set dial till 28 comes under cursor.

Read answer (just over 97 and which we judge as 972) on Scale No. 1 under datum.

By visual inspection it will be seen that the answer must be in the neighbourhood of .09. Therefore we write our answer as given by the Calculator as .0972.

Worked out on the Long Scale the procedure is as follows :-

Set 347 (midway between the marks 346 and 348, 6th circle from centre) under the datum.

Set cursor to Unity line.

Set dial till 28 (5th circle from centre) comes under

Read answer a shade over the 3rd graduation past the 97 mark

This we should estimate as 9716, and from the considerations mentioned when worked out on the Short Scale we should call it 0.09716, an answer correct to five figures

Multiplication of Three Factors on the Short or Long Scale

The method is precisely the same whichever scale is used so it will be described only for the Short Scale. Ex. 3: Find the product of $.0347 \times 2.8 \times 63.5$.

Set 347 on Scale No. 1 under datum.

Set cursor to Unity line

Set dial till 2.8 on Scale No. 1 comes under cursor.

All the above settings as shown in Ex. 2.

Set cursor to Unity line

Set dial till 635 on Scale No. 1 comes under cursor. This is the 7th division past the 6 mark.

Read answer 6:17 on Scale No. 1 under datum.

By actual multiplication the correct answer is 6.16966. showing a close approximation by the use of the Short

Scale of the instrument Had the Long Scale been used we should have found

the answer to come just a shade under 617 mark and put the answer down as 6.1695, a still better approximation. Multiplication of Four or more Factors on the Short

or Long Scale.

Ex. 4: Find the product of $.0347 \times 2.8 \times 63.5 \times 4.9$. The method is precisely the same for the first three

factors shown in Ex. 3 above when we had a reading of 617 (approx) under the datum.

We now again set the cursor to Unity and then set dial till 49 comes under the cursor.

Read product $.0347 \times 2.8 \times 63.5 \times 4.9$ under datum.

This, if using the Short Scale, comes just a little over the first division past the 3, and we should estimate the answer as 30.23 an approximately exact answer.

Ex 5: Divide 7256 by 13.85. Set 7256 under the datum.

(On the Short Scale this number would be represented as nearly as possible at a point a little over the 5th graduation mark following the 7. On the Long Scale it would be at a point a shade beyond the first short graduation mark past 725. The first short graduation mark, of course, represents 7255.)

Now set the cursor to 13.85.

On the Short Scale this is at a point midway between the 8th and 9th graduation past the 13. On the Long Scale it is at a point on the 2nd circle from the centre midway between the 2nd and 3rd graduation following

Turn dial till the Unity line comes under the cursor Read answer 524 on the Short Scale No. 1 and 523.9 on the Long Scale, this latter figure being quite correct

FRACTIONS

Consider first a fraction with two factors in the numerator and one in the denominator, and worked out on the Short Scale

676 · 9 × 364 Ex. 6: Solve

Set dial till 6769 comes under the datum

Set cursor to 1149

Set dial till 364 comes under cursor

Read answer 2157 under datum

The correct answer by actual multiplication and division is 2157.5. When worked out on the Long Scale the answer came barely 2158 and we should therefore

put it down as 2157.5, a correct result

It will have been noticed in working out the previous examples on the Long Scale, that greater initial accuracy of setting can be got than when the Short Scale is used. and thus it is that a more accurate result is obtained when reading the answer. The method of locating the circle on which to read the answer which is given on p. 5-6 should however always be borne in mind. The principle of working is, however, precisely the sama when either scale is used, and to avoid confusion the succeeding examples will all be worked out on the Short Scale.

Consider now fractions with several factors in the numerator and denominator. See notes on page 7

Ex. 7: Solve
$$\frac{19.5 \times 66.6 \times .0042}{8.9}$$

Work this as $\frac{19.5 \times 66.6 \times .0042}{8.9 \times 1}$ taking the factors alternately from the numerator and the denominator.

Set 19.5 under datum

Set cursor to 8.9.

Set 66.6 to cursor. Set cursor to Unity.

Set 42 to cursor.

Read answer . 613 under datum the decimal point being fixed by a rough mental calculation as indicated in examples which follow.

The correct answer worked out by actual multiplication and division is .61287

13695...6pr OS & Latin...14 ems...THREE...

Ex. 8: Solve $\frac{13.8 \times 120}{15.8 \times 176 \times 2.42}$ Work this as 13.8 × 723.6 × 1 × 1 $15.8 \times 176 \times 2.49$

taking the factors alternately as in the previous example. Set 13.8 under datum

Set cursor to 15.8 Set 7236 to cursor Set cursor to 176 Set Unity to cursor Set cursor to 249

Set Unity to cursor.

Read answer 1.483 under datum.

The correct answer is 1.48388 (a close approximation). EXERCISES WITH THE RECIPROCAL SCALE No. 2

If the reader has followed the previously worked out examples carefully he will be in a position to solve in a routine way any compound fraction presented to him, and also to apply the more rapid method of multiplication and division permitted when Scales Nos. 1 and 2 are used in conjunction and which will now be explained

Multiplication of an ODD number of Factors using Scales Nos. 1 and 2 in conjunction.

Ex. 9: Find the product of $8.42 \times 16.16 \times .422$

Set 842 on Scale No. 1 under datum. Set cursor to 1616 on Scale No. 2. Set 422 on Scale No. 1 under cursor

Read answer 57.4 on Scale No. 1 under datum.

By actual multiplication the correct answer is 57.42036. The decimal point is fixed mentally in this way: $\cdot 422$ is roughly $\cdot 5$; $\cdot 5 \times 8 \cdot 42$ is roughly 4, and 4 × 16.16 is roughly 56. Therefore there are two whole numbers in the answer.

Ex. 10: Find the product of $\cdot 354 \times 29 \cdot 4 \times 63 \cdot 6 \times \cdot 862$

(4 factors).

This will be worked as $\cdot 354 \times 29 \cdot 4 \times 63 \cdot 6 \times \cdot 862 \times 1$ to make it into an odd number of factors, i.e., 5.

Set 354 on Scale No. 1 under datum. Set cursor to 294 on Scale No. 2.

Set 636 on Scale No. 1 under cursor. Set cursor to 862 on Scale No. 2.

Set Unity on Scale No. 1 under cursor.

Read answer 571 on Scale No. 1 under datum (5 movements).

The correct answer by actual multiplication is 570.578 The decimal point is fixed mentally in this way: :354 is roughly one-third; one third of 29.4 is roughly 9; 9 times 63.6 is roughly 560: 560 multiplied by 8 is roughly 500. Therefore the answer must contain three whole numbers and is 571.

It is interesting to compare the above example with the 9 movements necessary when using either Scale Nos. 1 or 4 alone, or by comparing it with the movements of an ordinary Straight Slide-Rule with its intermittent "end-switching." This is only one of many illustrations that could be given

Rapid Division with Scales Nos. 1 and 2 used in conjunction, with EVEN number of Factors in the Denominator

Ex. 11: Find the value of $\frac{6734}{9.6 \times 142.5}$ where there is

an EVEN number of factors in the denominator. Set 6734 on Scale No. 1 under datum.

Set cursor to 96 on Scale No. 2 Set 1425 on Scale No. 2 under cursor.

Read answer 4.92 on Scale 1 under datum (3 movements), the position of the decimal point being fixed mentally as explained above. The correct answer worked out by multiplication and division is 4.923.

4276 Ex. 12: Solve $3.42 \times 18.7 \times 32.62$

Here the artifice may be adopted of inserting an extra factor, 1, into the denominator to make it coutain an even number of factors, thus:

3 · 42 × 18 · 7 × 39 · 69 × 1 Set 4276 on Scale No. 1 under datum.

Set cursor to 342 on Scale No. 1. Set 187 on Scale No. 2 under cursor.

Set cursor to 3262 on Scale No. 1. Set 1 (Unity) under cursor.

Read answer 2.050 on Scale No. 1 under datum (5 movements).

Further exercises with the Reciprocal Scale.

Ex. 13: Find the decimal equivalent of $\frac{1}{6.456}$

Set cursor over 6456 on Scale No. 1. Read under

cursor on Scale No. 2 0.1548.

In setting cursor to 6456 on Scale No. 1, we note that there are 20 graduations between 6 and 7, the reading advancing clockwise 6.05, 6.10, 6.15, 6.20, etc., and 6.456 is between 6.4 and 6.5 its exact position being estimated. Conceive this space to be divided into 100 parts, and advance 56 of these parts past 6.4. i.e., just a little more than half-way.

Reading Scale No. 2 anti-clockwise, we make the value under the cursor as near as may be 1548.

From inspection of the fraction its value is obviously between one-sixth and one-seventh, and without hesitation write down the decimal value as 0:1548

Ex. 14: Find the decimal equivalent of \(\frac{1}{8475}\)

Set cursor over 3475 on Scale No. 1. Read 2878 on Scale No. 2 under cursor.

The fraction is manifestly less than expressed decimally will require 3 cyphers after the decimal point so we write the answer 0.0002878.

In setting 3475 under the cursor we note it falls between the graduations 34 and 35, and that between 34 and 35 there are 5 graduations, each advancing 2, thus: 340, 342, 344, 346, 348, 350. Half-way between 346 and 348 is 347 and a shade past this is 3475.

Reading Scale No. 2 the cursor is just short of the value 288. We estimate it as 2878 and the answer, therefore, is 0.0002878.

Ex. 15: Find the decimal equivalent of $\frac{1}{0.0284}$

Set cursor over 284 on Scale No. 1. (This is the fourth graduation line past the 28 mark.

The reading on Scale No. 2 under cursor is just past the graduation following the 35 mark, reading anticlockwise, and where each space counts 2. We estimate it as 3521.

By inspection the value of the fraction is seen to be more than $\frac{1}{20}$ and we write down the answer as $35 \cdot 21$.

The three preceding examples are good illustrations of the care required in Scale reading; in noting the value of the graduations, and whether the advance of the Scale is clockwise or anti-clockwise. Scale No. 2, the Reciprocal Scale, it may be noted, is the only one graduated anti-clockwise.

Ex. 16: Find the decimal value of $\frac{1}{25}$

Set 37 on Scale No. 1 under cursor. Read .027 on Scale No. 2 under cursor.

Note that in reading decimal values of fractions less than one-tenth there will be one cypher placed after the decimal point, and preceding the number as read from the Reciprocal Scale. With values less than onehundredth and greater than one-thousandth two cyphers will precede the number and so on.

Ex. 17: Find the fractional value of .1428 Set (anti-clockwise) 1428 on Scale No. 2 under cursor.

Read 7 on Scale No. 1 under cursor Fractional value is therefore -.

Ex. 18: Find the fractional value of .00653 Set 653 on Scale No. 2 under cursor. Read 153 on Scale No. 1 under cursor.

Fractional value is therefore

Note that as many cyphers must follow the 153 as there are cyphers following the decimal point in the given number

xamples of Powers (Involution) using Reciprocal Scale. Ex. 19: Find the value of $(36.7)^2$

This can be done with the Calculator in two ways. either by multiplying 36.7 by itself as an ordinary multiplication sum, as previously described, or by the method shown below using the Reciprocal Scale.

Set 36.7 on Scale No 1 under datum. Set cursor to 36.7 on Scale No. 2 Turn dial till 1 comes under cursor. Read 1347 under datum on Scale No. 1. Ex. 20: Find the value of (16.4)3

This can be done by extended multiplication, 16.4 × 16.4 × 16.4 on Scale No. 1, or by the method shown below, in which we first find the square of 16.4, as in Example above, and then multiply this result on Scale No. 1 by 16.4

Thus set 16.4 on Scale No. 1 under datum. Set cursor to 16.4 on Scale No. 2.

Turn dial till 16.4 on Scale No. 1 comes under cursor. Read 441 under datum on Scale No. 1.

Note.—The result is obtained in 3 movements.

The use of the Reciprocal Scale also makes possible the calculation of expressions such as (.310)2.1 or (.496).5, etc., without the cumbersome method of ordinary logarithms, i.e., of having a negative characteristic, which must be made exactly divisible by the index. When quantities less than unity are being dealt with, we work on the Reciprocal Scale No. 2 instead of the outer Multiplying Scale No. 1, as shown below in the solution of the above examples.

Ex. 21: Find the value of $(0.310)^{2.1}$

Because 0.310 is less than unity, and answer will be also set 0.310 on Reciprocal Scale and read its log. on \log scale = 0.5085 (approx.).

Multiply by 2·1 and take wholly negative since answer will be less than unity

Then log. of answer = $-(2 \cdot 1 \times 0.5085) = -1.0678$ (using Scale Nos. 1 or 4 for this multiplication, preferably the latter).

Set decimal part (.0678) on log. scale, and read answer on Reciprocal Scale which = 0.0856

Ex. 22: Find the value of $(0.496)^{0.5}$

Because 0.496 is less than unity, and answer will be also, work on Reciprocal Scale and Log Scale

Set 0.496 on Reciprocal Scale, and read its log. (0.305)

on Log. Scale No. 3.

Multiply by 0.5 and take wholly negative since answer will be less than unity.

Then log. of answer = $-(0.5 \times 0.305) = -0.1525$. Set 0.1525 on Log. Scale, and read answer on Reciprocal Scale = 0.704.

LOGARITHMS

The value of logarithms to three and often four figures can be obtained from the Calculator by means of Scales Nos. 1 and 3. Limitations of space prevent any lengthy exposition of the theory of logs. but the following notes are useful.

The logarithm of a number is composed of two parts,

the Characteristic and the Mantissa.

The characteristic is the part of the logarithm to the left of the decimal point, and may be positive or negative.

If the number is greater than unity the characteristic is positive and one less in value than the number of figures to the left of the decimal point in the number.

If the number is less than unity the characteristic is negative and one greater than the number of cyphers to the right of the decimal point in the number. The indication of negative value is shown by a minus sign over the top of the characteristic.

The Mantissa is the part of the logarithm to the right of the decimal point and is always positive, and for the same figures always the same wherever the decimal

point may be.

These features of logarithms are shown in the following examples:—

Log of 278 is $2\cdot444$. Log of $0\cdot278$ is $\overline{1}\cdot444$. , $27\cdot8$, $1\cdot444$. , $0\cdot0278$, $\overline{2}\cdot444$. , $0\cdot0278$, $\overline{3}\cdot444$. , $0\cdot00278$, $\overline{3}\cdot444$.

(A fuller description of logs., with tables of logs, and anti-logs., will be found in "Fowler's Machinists Pocket Book," Scientific Publishing Co., 316, Manchester Road, West Timperley, near Manchester, 3/9 net, post paid.)

Ex. 23: Find logarithm of 2675

Set cursor over 2675 on Scale No. 1. Read Mantissa of log., viz., 4273 on Scale No. 3. As there are four figures in the number 2675 all to the left of the decimal point, the characteristic of the log. is positive and its value is 3.

The complete log. therefore is 3.4273.

Ex. 24: Find logarithm of 50.75.

Set cursor over 5075 on Scale No. 1 (it lies between the third and fourth graduation line after 5). Read Mantissa of log. on Scale No. 3, viz., 7055

The characteristic of the log. (as there are two figures to left of decimal point) is 1

The complete log, is 1.7055.

Ex. 25: Find logarithm of 0.024076.

Set cursor over 24076 on Scale No. 1. This lies about one-third of the way between 24 (which represents 240) and the first graduation after it, which represents 2402. Read Mantissa of log, on Scale No. 3.

We make the reading 3815

As the number is less than unity, the characteristic is negative and as there is a cypher to the right of the decimal point its value is 2.

Therefore the logarithm of 0.024076 = 2.3815

To find a number corresponding to a given logarithm. Set the decimal portion of the logarithm on Scale No. 3 under the cursor and read on Scale No. 1 the corresponding number.

The index of the logarithm increased by 1 will be the number of integers in the given number when it is a whole number or the index diminished by 1 will be the number of prefixed cyphers when the number is a decimal fraction, and the index consequently negative.

Ex. 26: Thus given 2.1880 as the logarithm of a number. To find this number set 1880 (the decimal portion of the log.) on Scale No. 3 under cursor.

Read under cursor on Scale No. 1, 1542.

As 2 is the characteristic of the number there will be three whole figures to the left of the decimal point in the answer, which is therefore 154.2.

Ex. 27: Find the number which has 4.5250 as its logarithm.

Set 5250 (the decimal portion of the log.) on Scale No. 3 under cursor.

Read under cursor on Scale No. 1, 335.

As the index is 4, diminishing this by 1 in an arithmetical sense gives 3 as a remainder, but increasing 4 by 1 in an algebraical sense gives 3, and 3 is just the number of cyphers after the decimal point of the

number required, which thus becomes .000335

In the algebraical sense therefore the index is in every case increased by 1 to give the number of integers or

prefixed cyphers.

As explained at the commencement of this chapter Logarithms of Numbers can readily be obtained to three and often four figures by the use of Scales Nos 1 and 3 used in conjunction, but a method was evolved some years ago by one of our Calculator users—the late Mr. Harold Palmer, M.P.S., of Cheltenham-whereby the length of the log, scale may theoretically be increased from 11.75 inches (approx.) to 117.5 inches (approx.) i.e., ten times its natural length, and the mantissa beread to four decimal places definitely, and often to five Both logs, (mantissa portion) and anti-logs, are read on the Long Scale

A few examples showing the method adopted are given below.

Ex. 28: Find the log. of π (= 3.1416).

(1) Place the cursor over the value of π on the Long Scale. This is at a point about three-quarters of the first division past the 314 mark. This value is on the fifth circle from the centre. Deduct 1 from this circle number. Then 5-1=4

(2) Read the value on the Log. Scale No. 3 under

cursor = 9715.

Write down the result as 4.9715.

(4) Now divide 4.9715 by 10 = .49715The log. of 3.1416 is therefore .49715 (correct to five

decimal places). Ex. 29: Find the log. of 437.5.

(1) Set cursor to 437.5 on the Long Scale It appears on the 7th circle. Then 7 less 1 = 6.

(2) Read the value on the Log. Scale = .4098. (3) Combining the two values as before 6:4098 and dividing by 10 the mantissa becomes .64098 The complete log. is therefore 2.64098. (A very accurate

result, as, from tables, the value is given as 2.6409781.) It is surprising how quickly the process can be mentally effected, the dividing by 10 being a simple matter by altering the position of the decimal point. and the memorising of the subtraction of 1 from the

circle on which the number whose log, has to be found is placed.

Ex. 30: Find the log. of 1455.

(1) Place the cursor over 1455 on the Long Scale. This number appears on the 2nd circle. Then 2-1=1.

Read value on the Log. Scale = .628. Combining the two values 1 + .628 = 1.628 and dividing by 10, the mantissa becomes . 1628,

(4) The complete log is therefore 3.1608

It is, of course, possible, by reversing the previous process to find the antilog, of a number.

Fix. 31: Find the antilog, of 2.63205. (1) Multiply the mantissa by 10 = 6.3205

Turn cursor to .3205 on the Log. Scale . (3)

The result is found on the 6 + 1 = 7th circle = 4286

Note.—There is never any doubt as to which circle to read the antilog, on: and, in this case as the characteristic is 2, the answer becomes 428.6

Hyperholic Logarithms. - These which are to the base e = 2.71828 are much used in calculations relating to the expansion of gases. They also find extended use in higher mathematics. They can easily be derived by multiplying the common logarithm obtained from Scales Nos. 1 and 3 by 2.30258. The exact position of this multiplier denoted by log., 10 will be found as a gauge point on the outside of Scale No. 1.

Ex. 32: Find the hyperbolic log. of 14.35. First find common log, of 14.35.

Set cursor over 1435 on Scale No. 1.

Read mantissa of common log., viz., 1570 on Scale No. 3.

As there are two figures to the left of the decimal point in the number, and the number is greater than unity, the characteristic is 1, and positive,

Therefore the log, of 14.35 is 1.1570.

Now multiply 1.1570 by log.e10.

Set log. 10 on Scale No. 1 under datum.

Turn cursor to 1, and then turn dial till 11570 comes under cursor.

Read answer 2665 = hyperbolic log, under datum. To convert hyperbolic logs, to common logs, multiply by 0.43429 indicated as log roe in the gauge points.

EXTRACTING ROOTS BY LOGARITHMS

Roots of numbers as for example 2/162 or 3/9176 can easily be extracted by means of logarithms. The small (2) and (3) of the root sign is called the index of the root, and any sort of a number may be found by dividing its logarithm by this index. The quotient obtained is then the logarithm of the root.

Ex. 33: Find 3 1/694

Set cursor over 694 on Scale No. 1, and read mantissa of log, on Scale No. 3, viz., .8414. As there are 3 whole numbers in 694 the characteristic will be 2 and the log, of 694 will therefore be 2.8414.

 $2.8414 \div 3 = 0.9471$

Hence $\log \frac{3}{4} / 694 = 0.9471$

Set 9471 on Scale No. 3 under cursor and read on Scale No. 1 885. By inspection we should say that 3/694 is therefore 8.85.

Ex. 34: Find $\sqrt[5]{0.82}$

Set cursor over 82 on Scale No. 1. Read on Scale No. 3 mantissa of log. = .9138.

Therefore log. 0.82-= 1.9138.

In this case it is not possible to divide directly by 5 because there is a negative characteristic, and a positive muntissa. The artiface of adding numerically as many negative units, or parts of units, to the characteristic as is necessary to make it evenly contain the index of the root is adopted. The same number of positive units or parts of units is then added to the mantissa, and each is then separately divided by the index.

Thus $\overline{1} + \overline{4} = \overline{5}$ and $\overline{5} \div 5 = \overline{1}$.

 $\cdot 9138 + 4 = 4 \cdot 9138$ and $4 \cdot 9138 \div 5 = \cdot 9827$

 $\overline{1} + .9827 = \overline{1.9827} = \log. \sqrt[5]{0.82}$

Therefore $\sqrt[5]{0.82} = 0.9610^{\circ}$ obtained by setting 9827 on Scale No. 3 under cursor and reading this value 0.9610 on Scale No. 1.

Ex. 35: Find 2.4 1/0.6

Set cursor over 6 on Scale No. 1 and read on Scale No. 3 mantissa of log. = 778.

Characteristic will be 1. Therefore log, $0 \cdot 6 = \overline{1} \cdot 778$. If we add $(-1 \cdot 4)$ to the characteristic of this log, it will be evenly divisible by the index of the root (viz., $2 \cdot 4$). Hence $\overline{1} + (-1 \cdot 4) = -2 \cdot 4$ and $-2 \cdot 4 \div 2 \cdot 4 = \overline{1}$. Now add $+ 1 \cdot 4$ to the mantissa portion of log, and divide this by the index of the root.

Thus .778 + 1.4 = 2.178 and $2.178 \div 2.4 = .9075$.

Adding these together we get 1-9075.

Therefore $\log_{\bullet}^{2.4}\sqrt{0.6} = 1.9075$. Now set 9075 on Log. Scale No. 3, and read

 $\frac{2\cdot4}{\sqrt{0.6}}$ on Scale No. 1 = 0.808.

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extracting Square and 4th Roots by means of Scales

Nos. 1 and 2 used in conjunction.

Ex. 36: Find the square root of 1849.

Set 1849 on Scale No. 1 under datum.

Set cursor to 1.

Turn dial anti-clockwise until the same number comes simultaneously under the datum on Scale No. 1, and the cursor on Scale No. 2. This number is 43 the square root of 1849. Opposite 43 on either Scale No. 1

or Scale No. 2 will be found the value of $\frac{1}{\sqrt{1849}}$, viz.: 0-02325.

It will be observed that two values of square root may be obtained in this way. For instance in above example we can get either 43 coming on Scales Nos. 1 and 2, when the unity line on the dial comes opposite the mid-point between datum and cursor, or we could get 136 when the unity line falls midway between the datum and cursor.

The other value, for example the 136 given above, is the square root of the original number (1849) multiplied by the square root of 10

Thus $136 = \sqrt{1849} \times \sqrt{10}$

Ex. 37: Find the 4th root of 1849.

Proceed as in Example 36 above to find the square root (43) and then obtain the square root of 43 in a similar manner.

Set 43 on Scale No. 1 under the datum.

Set cursor to 1.

Turn dial anti-clockwise until the same number comes simultaneously under the datum on Scale No. 1 and the cursor on Scale 2. Thus 6.56 is the 4th root of 1849. Obtaining Powers of Numbers by Logarithms.

To raise a number to a given power multiply the logarithm of the number by the power index; the product is then the logarithm of the result.

Ex 38: Find the value of 27.53

Here the log, of the result will be 3 times the log, of 27.5.

Set cursor over 27.5 on Scale No. 1.

Read log. on Scale No. 3 (\cdot 4393 and add characteristic (1) = 1 \cdot 4393

 $3 \times 1.4393 = 4.3179$

Set cursor over ·3179 on Log. Scale No. 3.

Read 2079 on Scale No. 1 under cursor.

Since the characteristic is 4 we must add another cypher to this result to make 5 figures to the left of the decimal place. The answer is therefore 20790.0.

The correct arithmetical answer is 20796 · 875.

Ex. 39: Find the value of 161.33

Set cursor over 16 on Scale No. 1 and read log. on Scale No. 3 adding characteristic (1) to same. This equals 1.204.

Multiply 1.204 by 1.33 on Scale No. 1.

Set 1.204 to datum.

Set cursor to unity.

Set dial till 1.33 comes under cursor.

Read $1.204 \times 1.33 = 1.60$ on Scale No. 1.

Set 60 on Log. Scale No. 3 under cursor. Read 3981 on Scale No. 1 under cursor.

Since characteristic will have 2 figures to left of decimal point the answer i 39.81.

EXAMPLES SHOWING THE USE OF "GAUGE POINTS" MARKED ROUND OUTSIDE OF SCALE No. 1

Ex. 40: How Many yards are there in 396 metres?

Set cursor to 1

Turn dial till "Metres to Yards" comes under cursor. Read answer 433 on Scale No. 1 under datum.

Ex. 41: How many metres are there in 660 vards?

Set 660 on Scale No. 1 under datum.

Set cursor to 1

Turn dial till "Yards to Metres" comes under cursor.
Read answer 603 on Scale No. 1 under datum.

Ex. 42: How many lbs. are there in 86 kilogrammes?

Set 86 on Scale No. 1 under datum.

Set cursor to 1.

Turn dial till "Kg. to lbs." comes under cursor. Read answer 189:3 on Scale No. 1 under datum.

Ex. 43: How many square kilometres are there in 43 square miles?

Set 44 on Scale No. 1 under datum.

Set cursor to Sq. Mile-Sq. Km.

Read answer 111.4 on Scale No. 1 under datum.

These examples will serve to show how such conversions are made. Care should, of course, always be taken in setting, and reading results, especially with regard to sub-divisions, which must bear proportionate values to the prime numbers used.

The values of the Conversion Factors marked on the outer scale are as follows: Metres to yards, 1:09361: vards to metres. 0.9144: sq. metres to sq. vards, 1.19599; sq. vards to sq. metres, 0.83613; sq. centimetres to sq. inches, 0.155; sq. inches to sq. centimetres, 6.45159: miles to kilometres, 1.60934: kilometres to miles, 0.62137; kilogrammes to lbs., 2.20462; lbs. to kilogrammes, 0.45359; inches to centimetres, 2.54: centimetres to inches, 0.3937; sq. miles to sq. kilometres, 2.5899; sq. kilometres to sq. miles, 0.3861. "C" = 1.12838 and is a constant which when multiplied by the square root of the area of a circle gives its diameter. $\sqrt{2} = 1.41421$; $\sqrt{3} = 1.73205$; Log. e10 = 2.30258: Log.10 e = 0.43429: $\pi = 3.14159$; gE = 32.2 feet per second; gr = 9.81 metres per second; radian = 57.2958 degrees: E.H.P. = 746: $\pi/4$ = 0.7854.

MENSURATION OF CIRCLES

Ex. 44: Find the area of a circle $3\frac{1}{2}$ inches diameter. Area = $d^2 \times \pi/4 = 3 \cdot 5 \times 3 \cdot 5 \times 7854$. Set 35 on Scale No. 1 under datum.

Set cursor to 35 on Scale No. 2.

Turn dial till $\pi/4$ (gauge point on outer circle) comes under cursor

Read area 9.62 square inches on Scale No. 1 under datum.

Ex. 45: 1 ind circumference of a circle 9.3 inches diam. Set 93 on Scale No. 1 under datum.

Set cursor to 1

Turn dial till π (gauge point on outer circle) comes under cursor.

Read circumference $29\cdot 2$ under datum on Scale No. 1. This and following examples may, of course, be worked out on the Long Scale (No. 4) in a similar manner; the value of π (3·1416) as well as the diameter being taken on the Long Scale. This will give a closer approximation, viz. 29·22 nearly.

Ex. 46: Find the diameter of a circle whose area is

Dia. = $\sqrt{\text{Are i}} \times \text{C}$. C = 1·12838 and is marked as a gauge point on outer circle.

Set 227 on Scale No. 1 under datum.

Set cursor to 1.

Turn dial till same number (15·07) comes under datum on Scale No. 1 and under cursor on Scale No. 2. This is the square root of 227.

Turn cursor to 1.

Turn dial till "C" comes under cursor.

Read answer 17 under datum on Scale No. 1.

PROBLEMS IN PERCENTAGES

In speaking of percentages confusion often arises through inattention to the basis on which it is measured. If A's salary is £75 and B's £50 it would be true to say A's salary w s 50 per cent. greater than B's, and equally true to say that B's salary was 33 per cent. less than A's. The fact is o ly expressed in two different ways. There can be no misapprehension in any case if the quantity representing the 100 is made clear. Set the question as a problem in fractions thus:—

Ex. 47: In an examination 27 scholars pass 1st class, 35 2nd class, and 63 3rd class. Express the various

numbers as percentages of the whole.

Here 27 + 35 + 63 = 125 and this total must be regarded as a 100 base which has to be divided into three similar proportions. Therefore if x, v, z are the three percentages, we have the following relationships:

$$\frac{27}{125} = \frac{x}{100}$$
 and $x = \frac{100 \times 27}{125} = 21.6$ per cent,

$$\frac{35}{125} = \frac{y}{100}$$
 and $y = \frac{100 \times 35}{125} = 28$ per cent.

$$\frac{63}{125} = \frac{z}{100}$$
 and $z = \frac{100 \times 63}{125} = 50.4$ per cent.

For this class of problem the instrument is very convenient. Set 1.0 on Scale No. 1 under datum, and set cursor to 125 (i.e., 12.5). Rotate the dial until the several numbers 27, 35, 68 come under the cursor, and read the several percentages under the datum.

PROBLEMS IN PROPORTION

Set the question in simple fractional form as follows: Where A, B, C are certain known quantities and x is

the unknown quantity $\frac{A}{B} = \frac{C}{r}$

Each of these quantities may be in the numerator or denominator, as the operator finds it convenient in setting down their relationship, but must, of course, be done correctly. Then by C cross multiplication $A \times x = B \times C$

 $B \times C$ and $x = \frac{B \times C}{A}$

Ex. 48: If 15 men do a task in 28 days, in how many days will 21 men do it, assuming they work at the same rate. Obviously more men will take less time in the ratio of 15 to 21 and if x is the number of days, then

 $\frac{x}{28} = \frac{15}{21}$ and $x = \frac{28 \times 15}{21} = 20$ days.

This and the following example are worked out on the Calculator as previously described under "Fractions." Ex. 49: If a task takes 18 men 36 days, how many

me i will be required to do it in 27 days?

Obviously more men will be required in proportion to the increased speed at which the task must be done 36 × 18

therefore $\frac{36}{27} = \frac{x}{18}$ and $x = \frac{36 \times 18}{27} = 24$ men.

DISCOUNT

Ex. 50: What is the wholesale price of an article subject to a discount of 20 per cent., the retail price of which is 15%.

Set unity line to datum.

Set cursor to 15.

Turn dial to 80 (20 backwards) representing 20 per cent.

Read 12/- under cursor.

Profit on Returns.—Supposing a merchant can produce an article at 61d, per lb. and wishes to make a profit of

12½ per cent. on Returns the selling price can be determined as follows:—

Set unity line to datum.

Set cursor to $100 - 12\frac{1}{2} = 87.5$.

Then by setting any cost price under the cursor we can read the Selling Price under datum.

Thus $100 \times 6.125 = 7d$.

This can be worked out on either the Short or Long

Also while in this position we can place any Cost Price under the cursor and read the Selling Price (with 12½ per cent. profit) under the datum.

Thus cost $= 6\frac{3}{4}$ d. $(8 \cdot 0625$ d.) $= 8\frac{1}{16}$ d. $(8 \cdot 0625$ d.) $= 9 \cdot 175$ d. $(9\frac{3}{16}$ d.) $= 9 \cdot 5$ d. $(5\frac{1}{16}$ d.) $= 5 \cdot 5$ d. $(5\frac{1}{16}$ d.)

= 28d. (2/4d.)

 $= 24\frac{1}{2}d. (24.5d.)$

It is recommended that the Long Scale be used for these proble s as greater setting and reading accuracy can be obtailed thereby. If any other percentage of profit is required the working will be the same, but the percentage must always be subtracted from 100 and the cursor pliced to that figure.

Profit on Cost.—Assuming an article to cost $6\frac{1}{2}$ d. per lb. Find the selling price with the profit of $12\frac{1}{2}$ per

cent. on cost.

The working is as follows: $100 + 12\frac{1}{2} = 112\frac{1}{2}$.

Set 112.5 under datum on Long Scale.

Now by bringing any cost price on Long Scale under the cursor we get the selling price under the datum.

Thus $\cos t = 5d$. Selling price = $5 \cdot 625d$. Selling price = $8 \cdot 719d$.

= 233d. (23.75d.) = 26.75d.

The principle can be adapted to many uses.

For instance, if a merchant has an offer of 94d, per lb. and he can produce at say 84d, per lb. his profits on Returns will be obtained by calculating as follows:—(Use Long Scale.)

Set 91d. under datum.

Turn cursor to 8 Hd., i.e., 8.6875.

Turn dial till 100 (unity) comes under datum.

Read 91.5 under cursor.

The difference between datum and cursor will show the amount of profit on *Returns*, viz., 8½ per cent.

By turning 100 under cursor and reading 109.33 under the datum we read a profit of 9.33 per cent. on cost.

Again suppose the offer was for 9½d. per lb. and the

merchant wishes to make 12½ per cent. profit on Returns he will have to produce the goods at the following price.

Set 100 (unity) under datum.

Turn cursor to $100 - 12\frac{1}{2} = 100 - 12 \cdot 5 = 87 \cdot 5$. Now by settig the offered price, viz. $9\frac{1}{2}$ d., under the datum the figure which comes under the cursor will be the price at which the goods must be produced.

The answer is $8.\overline{1}875 = 8\frac{3}{16}d$. = Production Price. If an offer of $9\frac{1}{16}d$, per lb, is made, and a profit of

121 per cent. on cost is required the rule is:

Set 112½ (112:5) under datum. Turn cursor to 100 (unity).

Set price offered under datum.

Read Production Price under cursor = $8.4375 = 8\frac{7}{16}d$.

HINTS ON CALCULATIONS

Simplifying a Decimal Quantity.—Regard should be made to the value of the terminal figures which are struck off. If it be desired to contract 15.647 to four significant figures, then 15.65 is nearer than 15.64 because 7 is nearer 10 than 1; but if the original number had been 15.642, then 15.64 would have been the closer approximation.

Fractional Value of Decimals.—A misconception of the fractional value of decimals sometimes causes mistakes, especially if there are cyphers between the decimal point, and the first digit. To avoid this, remember that when expressed as a fraction the number of cyphers in the denominator is the same as the number of figures after the decimal point in the number.

For example:
$$3.04 = 3\frac{4}{100}$$
 or $\frac{304}{100}$;
 $.96 = \frac{96}{100}$; $.002 = \frac{2}{1000}$;

Lecating Position of Decimal Point.—When factors containing decimals are multiplied by ordinary arithmetic there is no difficulty in locating the decimal point; one simply ticks off the same number of decimal figures in the answer as there are in the factors, but this cannot always be done when the operation is performed with a scale or scales. The number of significant figures in the answer (i.e., the number which can be written down) cannot be stated beforehand. It will depend on several things, viz., the size of the factors, the degree of sub-division of the scale, the accuracy of the scale or scales and the accuracy of the operator in setting and reading them.

The position of the decimal point, however, determines the accuracy of the answer and its location is therefore important.

A rough idea of the result of a calculation is often known beforehand, or if not, the position of the decimal point, where necessary, can be approximated by a rough survey of the fraction expressing the required calculation. There are rules, but they are more trouble to remember than they are worth, and it is better for the operator to rely on first principles and rapid mental arithmetic.

For example, suppose the value of the following were required:—

 $6.92 \times 746 \times 19.2 \times 9$

 2876×92.5

we could reason mentally, and roughly, as follows:— $6\cdot 9$ is practically 7, and 7 into 2,876 is roughly 400, 400 into 746 is roughly 2; 2 into 92·5 is roughly 45. This would be in the denominator, and for the numerator we should still have left $19\cdot 2\times 9$, roughly 170. This divided by 45 would obviously give a value less than 10. In putting down the answer, therefore, we should write all figures after the first one to the right of the decimal point. A rough estimate like this occupies less time to make than to describe, and is safer than any cut-and-dried rule.