

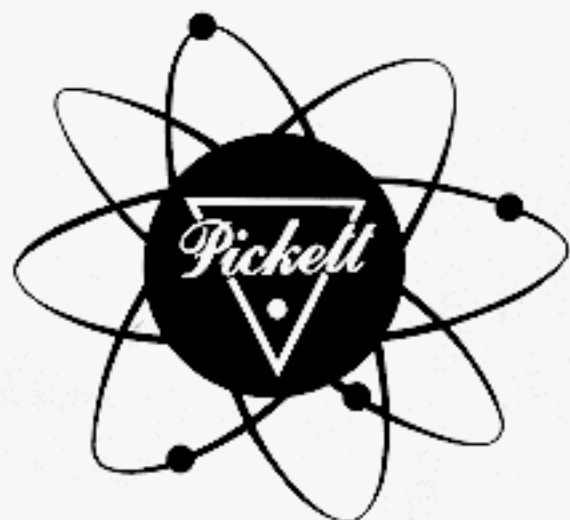
A teaching guide for slide rule instruction

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Chicago 5 / Alhambra, California



Introduction

Many teachers have not had extensive experience in teaching the slide rule. They are looking for suggestions for effective ways of introducing the slide rule to their students. Some teachers who have had much experience may nevertheless be interested in checking their own methods against those that others have found effective. This Teaching Guide has been prepared to help both groups of teachers.

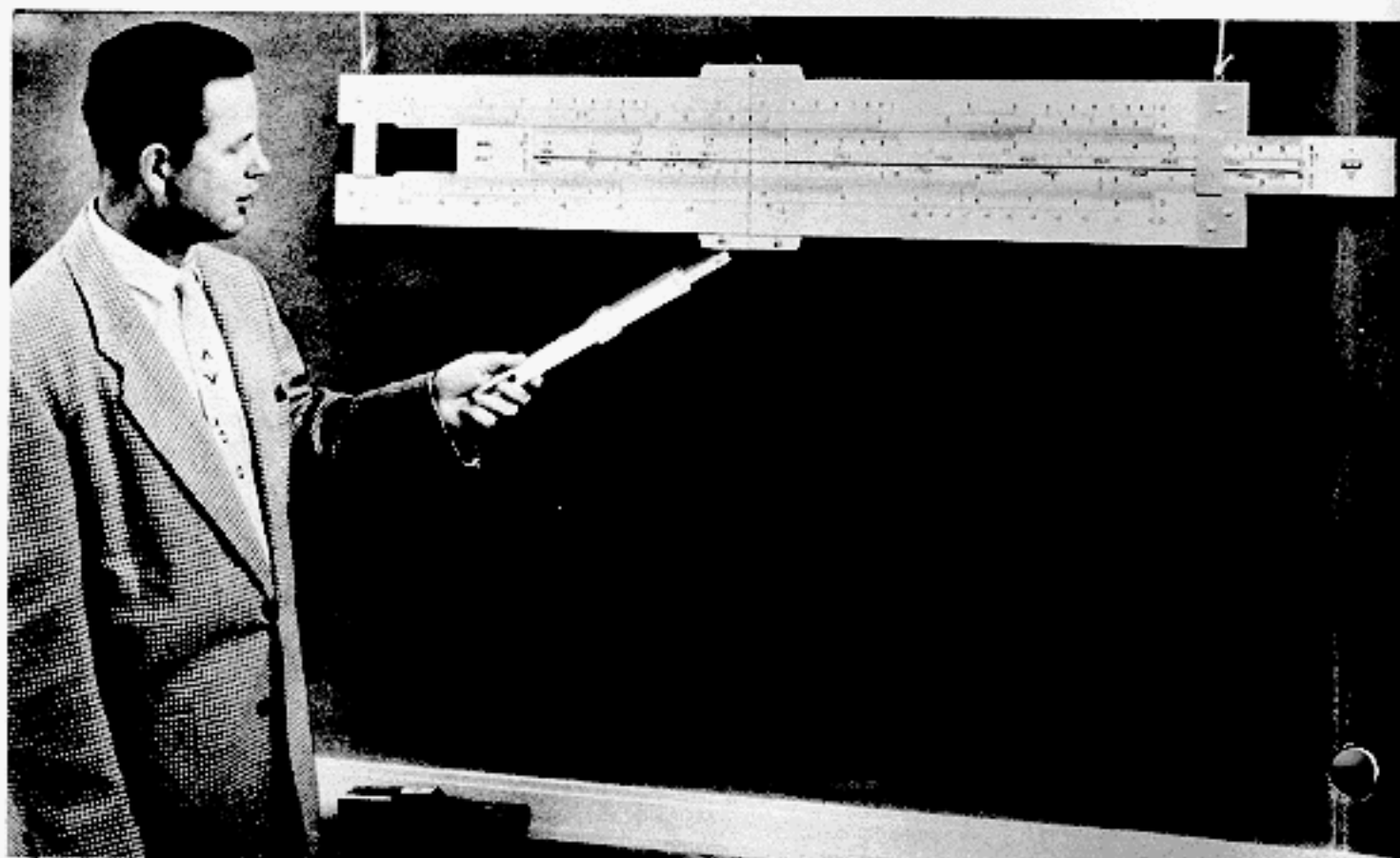
The students who are mentioned in the Guide are assumed to be in junior or senior high school classes. The Guide is organized into twelve sections. Some teachers may wish to follow the suggestions in each of these sections. Others will want to select only certain sections appropriate for their classes and time allotments. There is also some freedom of choice as to the order in which the various topics may be taken up. Comments on decisions as to selections and order are included in the Guide.

If each teacher who looks through this Guide finds only one suggestion that will help him make the study of mathematics more interesting and effective for his students, the preparation of the Guide will have been worth the effort.

The author of this Guide, Maurice L. Hartung, is widely recognized for his work in curriculum and methods in mathematics. He has written many texts and professional articles for this field. Since 1938 a member of the faculty of the University of Chicago where he holds the rank of Professor, Dr. Hartung received both degrees of M.A. and Ph.D. in Mathematics from the University of Wisconsin.

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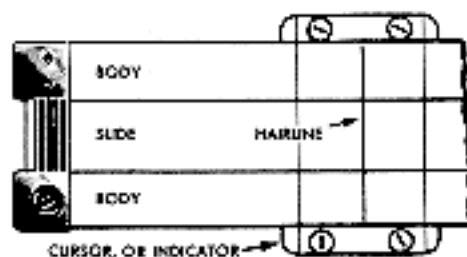
Introducing the slide rule

Motivation is no problem in the early stages of teaching students to use a slide rule. With the instrument in their hands they are ready and eager to begin using it. Tell the class that the slide rule is a fairly simple tool by means of which answers to involved mathematical problems can be easily obtained. To solve the problems easily and with confidence, however, it is necessary that the students have a clear understanding of the operation of the slide rule.

Begin by showing the students how to hold a slide rule. The ends of the rule should be held between thumb and forefinger. This will insure free, smooth movement of the slide and of the indicator. Holding the rule at the center hinders this free movement.

The slide rule consists of three parts: (1) body; (2) slide; (3) cursor, or indicator, with its hairline. The scales on the body and slide are arranged to work together in solving problems, and each scale is named by a letter (A, B, C, D, etc.) or other symbol. The hairline on the indicator is used to help read the scales and adjust the slide.

Figure 1



**Helping students
get the general idea**

1.

1.1 Call attention to the scales on the slide rule. Tell the students that the C and D scales are the most important scales, and that the basic use of these scales is to multiply numbers. Identify the body, the slide, and the cursor, or indicator, with its hairline. If a large demonstrator rule is available, set it to show $2 \times 3 = 6$; that is, set 1 on the C scale over 2 on the D scale. Set the hairline of the cursor over 3 on the C scale and show the class that the answer, 6, may be read on the D scale under the hairline. Have each student set his own rule and read the answer from it.

Move the cursor hairline to 4 on the C scale and show that $2 \times 4 = 8$ may now be read on the D scale.

1.2 Next, write an example such as 2.34×36.8 on the board. Tell the students that before they can do more difficult examples like this, they must learn how to read the scales and locate the position that corresponds to each number.

**Helping students
learn to read
the scales**

2.

2.1 The primary graduations

2.11 Tell the students there are a few "vocabulary" terms they should know to aid in talking about the procedure.

Call attention to the subdivisions of the D scale, and tell the class that each mark is called a graduation.

Figure 2



Set the cursor hairline over the left index of D. Tell the students this graduation is called the "left-hand index." Have them note the large numeral "1" under it. Next, have them observe the mark at the right, called the "right-hand index," which is also labeled "1". By moving the cursor, help the students locate the other major subdivisions labeled "2", "3", "4", etc. Explain that these ten marks are called "primary" graduations and represent the first digit in any numeral. Point out that these primary graduations are closer together as one moves from left to right. Tell the students that the D scale is an example of a non-uniform scale.

2.2 The secondary graduations

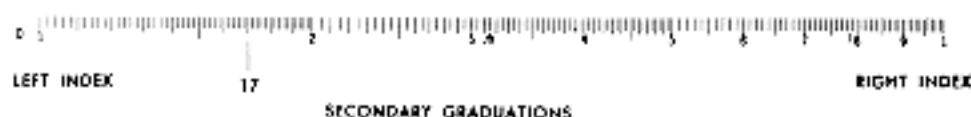
2.21 Next, have the students place the cursor hairline over the left-hand index, and move it slowly to the right till they come to the mark labeled with a small numeral "1", then continue through "2", "3", etc., to primary "2".* Tell the students these marks are called "secondary" graduations. Have the students notice that these graduations separate the interval between primary "1" and primary "2" into ten parts. Although these parts do not have the same length--the scale is non-uniform--the scale is read as though these were tenths. Have

*On 6-inch slide rules these graduations have no numerals beside them.

the students read the sequence between primary "1" and primary "2" as "1.1, 1.2, 1.3, 1.4, ... 1.9, 2.0." Write these numerals on the chalkboard in a horizontal line.

Continue to move the cursor hairline to the right over the other secondary graduations. Have the students read these aloud in sequence as: "2.1, 2.2, 2.3, ... 3.0, 3.1, 3.2, ... 4.0, 4.1, 4.2, ... 9.8, 9.9, 10.0." Call attention to the fact that in this sequence the right index is read as 10, not 1.

Figure 3



2.22 Now tell the students to move the hairline back to the left index and start over, but this time they are to read the left index as 10. Have them read the secondary graduations aloud in sequence, as: "11, 12, 13, ... 20, 21, 22, ... 30, 31, 32, ... 98, 99, 100." Also write this sequence of numerals on the chalkboard directly under the others (i. e., 1.1, 1.2, ...).

2.23 Repeat the cycle with the numerals: "110, 120, 130, ... 200, 210, 220, ... 300, 310, 320, ... 980, 990, 1000." Ask the students to study these sequences, and draw out for them the generalization that for each graduation the sequence of digits in the numeral is the same, but the position of the decimal point is changed.

2.3 The tertiary graduations

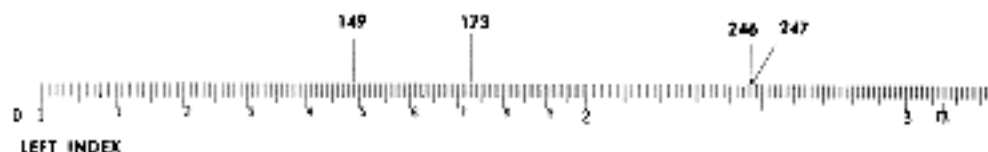
2.31 Some teachers may want to defer teaching the students how to read the third digit until after some work has been done on multiplication. This is a good plan because it gets the student "right into it" quickly. However, there are also advantages in going on immediately to further study of the scales. We assume at this point that the graduation scheme is that customary on 10-inch slide rules.

2.32 Have the students start by placing the hairline over the mark for 110. Call attention to the short graduations between secondary "1" and secondary "2". Have these read aloud in sequence as: "111, 112, 113, ... 119, 120." Continue with: "121, 122, 123, ... 129, 130, ... 198, 199, 200."

2.33 Now start back again at the left index, reading it as "100." Warn the students that the next reading is "tricky," and help them see that the sequence is: "101, 102, 103, ... 109, 110."

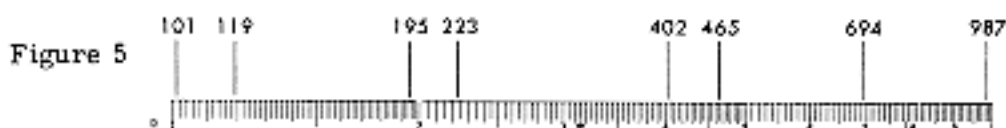
Move now to primary "2", and call attention to the fact that there are only five spaces between it and the next secondary graduation. Tell the students they can now "count by twos," reading the graduations in sequence as: "202, 204, 206, 208, 210, 212, ... 396, 398, 400."

Figure 4



2.34 Now have the students start back at 2, and tell them to place the hairline about halfway between the mark for primary 2 and the next mark. Tell them this position is read as "201." Move the cursor between "202" and "204," and read "203." Continue to read the graduations for odd numbers up to 399.

Some sample readings are shown in Figure 5.



2.35 A major psychological difficulty may arise at this point. Some students may be disturbed because they feel they cannot place the hairline precisely where it "should be." They become unduly concerned about error. At this time they are not quite ready for a fully satisfying discussion of this problem. It is best simply to point out that with practice they will learn to estimate with adequate precision where the hairline should be. For the present, they should try to place it just slightly to the right of the midpoint of the interval between two graduations. Point out that engineers, scientists, and others use slide rules regularly and find the accuracy adequate for much of their work. If time permits, the students should learn the basic principles of approximate computation. Some of the essentials will be discussed later in this teaching guide.

2.36 Next, have the students observe that to the right of primary "4" there is only one subdivision between each secondary graduation. Tell the students to read the graduations "by fives," thus: "405, 410, 415, ... 505, 510, 515, ... 605, 610, 615, ... 995, 1000." Between these graduations the students must estimate. Have them start at 400 and move the hairline slightly to show where 401 should be, then move again to 402, 403, 404, and thus arrive at 405. Continue in this way until they gain some confidence in "splitting" the interval. Point out, for example, that 413 will be nearer 415 than 410, and 414 will be very close to 415.

2.37 Finally, by returning once more to the left-hand index, explain how a fourth digit can be read in the interval between primary "1" and primary "2". Begin, for example, at 136, which can be located at a graduation. By splitting the next interval in half, show where 1365 is located. Continue with similar examples.

2.38 At this point the students should be given some practice in reading the C and D scales. The large demonstrator rule may be used to give them settings to read, and they should each write the numerals on paper. They should also make settings that correspond to given numbers. Prolonged practice is not wise, however. The most effective practice comes with actual computation, where the correct answer serves as a check on the settings.

Helping students understand the basic principles

Some teachers may wish to defer instruction on the basic principles until after the students have made more progress in learning to compute with the slide rule. Those teachers may turn at once to Section III. This section may be omitted entirely if only a limited amount of time is available. If, however, understanding of principles is a major objective (as it should rightfully be), time should be found for learning activities of the type described here.

Tell the students that the ordinary slide rule is used to multiply and divide numbers, and to do many other more complicated computations. Explain that they will understand better how it works if they take a few minutes to see how a slide rule for addition can be made.

Scales for addition

1.

1.1 Remind the students that they have used a foot-rule many times to measure lengths. Demonstrate how two foot-rules, X and Y, can be used together to add numbers.

For example, in Figure 6 the 0 of Rule X is placed opposite 3 of Rule Y. Under 4 of Rule X you see 7 of Rule Y. Used in this way,

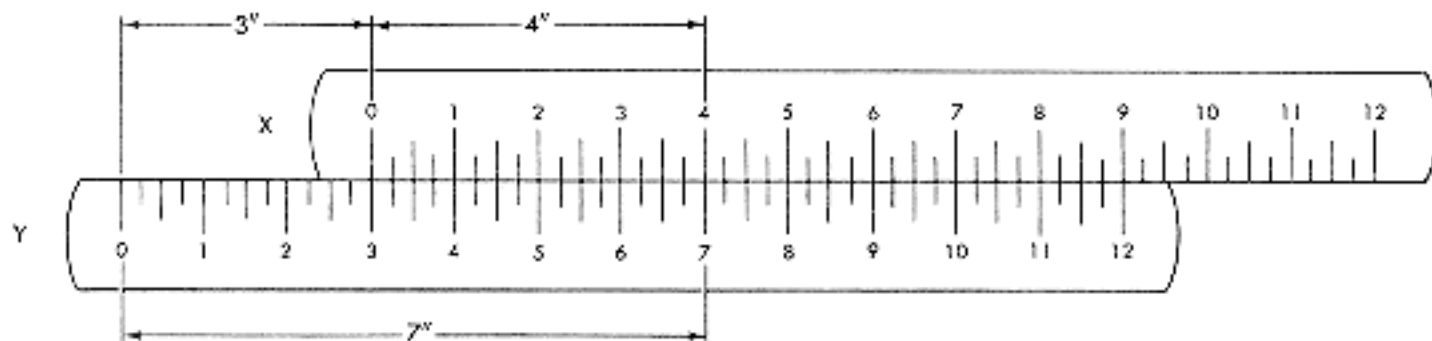


Figure 6

these rules provide a mechanical way of showing that $3 + 4 = 7$. At the same time, Figure 6 shows that $3 + 5 = 8$, and other sums. Also, by sliding Rule X along Rule Y, the 0 mark of Rule X can be placed opposite any mark on Rule Y. The two rulers used together become a slide rule for simple addition.

1.2 Point out that the marks (or "graduations") and numerals on a foot-rule are an example of a scale, and that the foot-rule is a uniform scale. Have the students themselves demonstrate addition using two ordinary foot-rules, or two uniform scales drawn on paper.

Explain that foot-rules are not often used to add as in Figure 6 because the scales are short. Only the numbers from 0 to 12 are represented. Facts such as $3 + 4 = 7$ are easy to remember, and no one needs a mechanical device to do such simple examples. Also, have them notice that the example $3 + 11$ causes trouble because the numeral 11 on the X scale is on the part that extends beyond the Y scale. There is no numeral 14 on a foot-rule.

1.3 Next, demonstrate how examples such as $3 + 11 = 14$ and $18 + 17 = 35$ can be done mechanically by using two yardsticks. The scale on a yardstick reads from 0 to 36. However, an example, such as $18 + 27$, causes trouble because the sum, 45, is not on the scale of a yardstick. Bring out through questioning that by using two longer scales this example can be done. Help students see that any addition example can be done by using scales that are long enough, but very long scales would not be convenient to carry around or use. To be convenient, a slide rule scale should not be much more than about 10 inches in length.

1.4 Now call attention to the fact that the scale on a foot-rule has marks to represent fractions. Usually the marks show eighths, but sometimes sixteenths and even finer subdivisions are made. By using these marks, demonstrate how examples such as

$$2\frac{3}{8} + 6\frac{5}{16} = 8\frac{11}{16}$$

can be done easily with sliding foot-rules. Finally, demonstrate how marks to represent decimal fractions, such as tenths, can also be used, as shown on the bottom scale of Figure 7. By using scales graduated in tenths, show how examples such as $2.3 + 6.9 = 9.2$ can easily be done. Help students see, however, that graduations to show hundredths would be too close together to read without a magnifier.

1.5 If "meter sticks" are available, as they usually are in science laboratories, have the students demonstrate addition of numbers

Figure 7



expressed to tenths and to hundredths by means of two adjacent meter sticks.

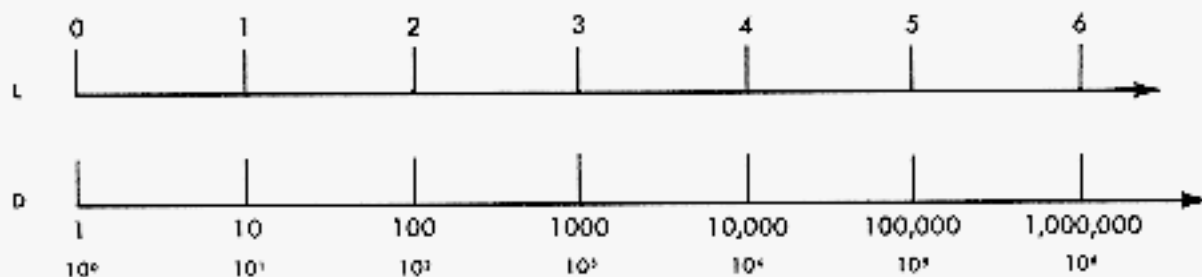
1.6 Tell the students they will learn how the ordinary slide rule overcomes the kinds of difficulties mentioned above. The difficulties arise because the scales must be short. If the scales could be as long as we pleased some things would be easier, but as a mechanical device the slide rule would not be the very convenient tool it now is.

Scales for multiplication

2.

2.1 Remind the students again that the scale of a foot-rule is uniform. The numerals 0, 1, 2, 3, etc., represent consecutive integers (sometimes called "whole numbers"). On a foot-rule, the distance between two graduations which represent consecutive integers is always one inch (see scale L of Figure 8). Have them make a non-uniform scale by putting marks on a line at intervals of 1 inch and by labeling the graduations as is done in scale D of Figure 8. The graduation labeled 0 on L is labeled 1 on D; the mark labeled 1 on L is labeled 10 on D; the mark labeled 2 on L is labeled 100 on D; etc. Tell the students that the marks for 2, 3, 4, etc., to 9 which would be between 1 and 10 on scale D need not be shown (see Figure 8). Also, the subdivision marks for 11, 12, 13, etc., to 99 which would be between 10 and 100 are not to be shown either. These 89 subdivision marks would have to be put in the same length (1 inch) as the 8 marks between 1 and 10. They would, therefore, be much closer together than they are on scale L.

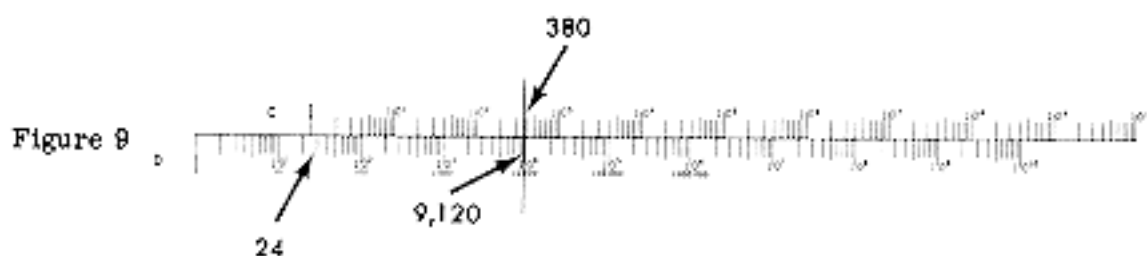
Figure 8



Similarly, the 899 subdivision marks for integers between 100 and 1000 are not to be shown (see Figure 8). These marks would be very much closer together than those with the same numerals would be on scale L. The graduations get more and more crowded as the numbers get larger. Tell the students scale D is a non-uniform scale, and this particular non-uniform scale is called a logarithmic scale.

2.2 Explain that these scales may be extended indefinitely. The only thing that limits the range is the amount of paper or other material you can use to put the graduations and numerals on. If you had a strip of paper a mile long, the range of scale L could be from 0 to 63,360. The corresponding numbers represented on scale D would then range from 1 to an enormous number. This number would have 63,361 digits

or figures in its numeral, and it would require a strip of paper about 440 feet long on which to type just this last numeral. Some idea of the size of this number is gained by remembering that it requires only ten digits to write "one billion" or 1,000,000,000. Tell the students that a slide rule which has two logarithmic scales (like scale D of Figure 8) can be used to multiply one number by another. Explain, for example, how to find 24×380 . Use a drawing like Figure 9 on the board. Explain that you would place the mark for 1 on one of the scales (labeled C in Figure 9) over the mark for 24 on the other scale. Then you would look for the mark at 380 on scale C. On scale D right under it you would find the answer 9,120.



2.3 As a summary, make clear that two logarithmic scales, if used just as the foot-rules were for addition, will provide the answers for multiplication examples. If the scales are long enough, the accuracy will be very great, and the position of the decimal point in the answer numeral can be seen. However, to be practical the scales must be short--only about 10 inches or less in length. Also, many of the subdivision graduations (such as for 24 and 38 in Figure 9) cannot be shown, so the user must learn to estimate where they would be.

Showing how short scales can be used

3.

3.1 Help the students recall that if we want to measure a distance of several feet, but have only a foot-rule, we measure by using the same scale over and over again. That is, we put the scale down and make a mark to show the first 12 inches, then slide the scale along and use it again. We repeat this as many times as necessary. In this way we can measure as much length as we wish with only a foot-rule, but we must count and keep track or remember how many times the scale was used. If necessary, demonstrate this by measuring a length of 3 or more feet with a foot-rule.

3.2 Then point out that, similarly, if we want to multiply any two numbers by using logarithmic scales, we can do it with short scales that show only the numerals from 1 to 10, but we must then keep track of the decimal point by some other scheme. In reality, we use the same single section or piece of scale over and over again. When we do this, we interpret the left-hand index as 1, or as 10, or as 100, or as 0.1, or as 0.01, etc., depending upon the number we are setting. Similarly, primary "2" may represent 2, or 20, or 200, or 0.2, or 0.02, or any

number whose numeral has the sequence of digits below:

... 000020000 ...

We disregard the location of the decimal point in the numeral.

3.3 For example, consider again the problem of finding 24×380 by using the C and D scales of an ordinary slide rule. First, the left-hand index of the C scale is set over 24 on the D scale. In this case the D scale represents the section between 10 and 100 of the complete logarithmic scale. The numeral "2" on the scale represents 20, and the numeral "3" represents 30. The fourth secondary subdivision between "2" and "3" represents 24.

Next, the hairline of the cursor is set over 380 on the C scale. In this case the C scale represents the section between 100 and 1000 of the complete logarithmic scale.

Help the students see that when the setting is made and read in this way it is the same as putting down the C scale once and saying "ten," then putting it down again and saying "one hundred," and then going on to 380 (see Figure 9). We are now in the section for "hundreds." The numeral "3" represents 300, and the numeral "4" represents 400. The eighth secondary subdivision between "3" and "4" represents 380.

Now when these settings are made on an ordinary slide rule, the hairline of the cursor will be just to the right of the numeral 9 of the D scale. Since we started at 24 of the D scale, we might think this should represent 90. However, because two lengths of C scale were used in reaching the "hundreds," these two lengths must also be accounted for on the D scale. The numeral 9 near the hairline must now be read as 9000 (see Figure 9). This example shows how a single section of the logarithmic scales can be used to get the answers if we know how to interpret the numerals.

Tell the students that in slide rule computation the decimal point in the answer is always found mentally or by some auxiliary method. Several such methods are discussed in a later section.

Teaching multiplication and division

Multiplication with the C and D scales

1.

The best procedure is to work out a few examples with the class (using the demonstrator slide rule) and then help the students generalize by formulating a "rule" such as is given in any manual on slide rule operation. The first examples should call for the left index of C, and the result should be readable on D and not be "off scale." The following are illustrative: 19.5×4.1 , 2.42×296 , 121×65 , 3.81×2.16 .

1.1 The first basic idea is that in using the ordinary C and D scales, decimal points in the numerals are ignored.* Thus, 2.34, 23.4, 0.0234, and every number with this sequence of digits is treated as 234. Remind the class that this is also customary in ordinary multiplication with paper and pencil.

1.2 The second basic idea the students should have is that multiplication by slide rule is accomplished by adding two scale lengths and reading the result on the D scale. (If the suggestions in Section II, parts 1 and 2, have been followed, the students will already know this. If the suggestions in part 3 have also been used, the students will understand it still better.)

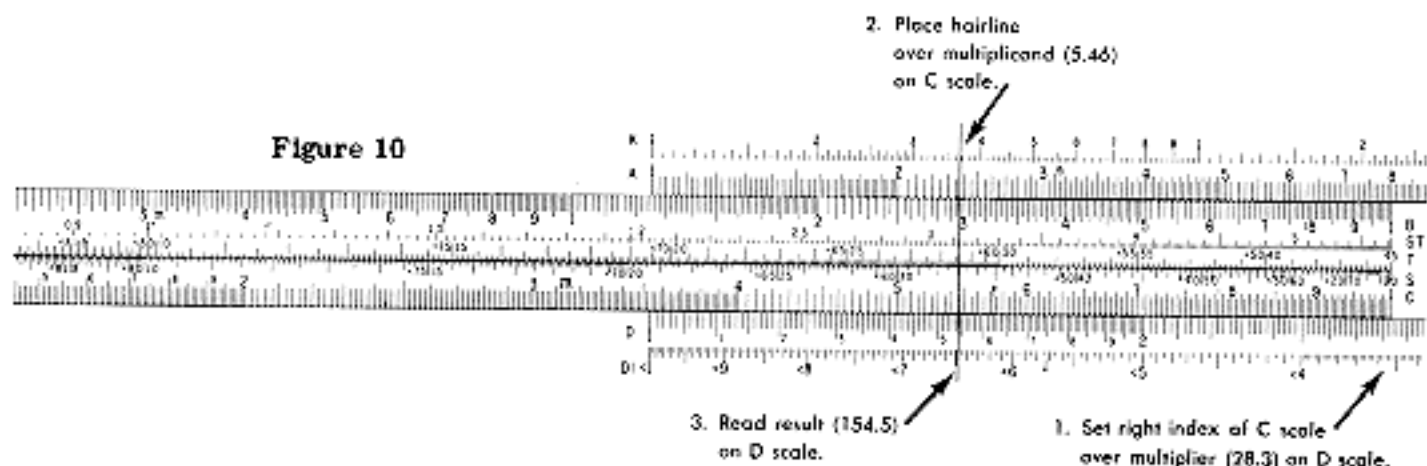
1.3 The third basic idea is that the location of the decimal point in the answer is an "extra step," just as it is in paper and pencil work. In

* If "powers of ten" or "decimal keeper" scales are used this statement is not true. See Section 6.

doing the first few examples, have the students estimate the result by rounding off the numbers and using mental computation. Encourage them to make this a habit and use it whenever possible in computation. The example 19.5×4.1 can be rounded to 20×4 . The students should see the result is near 80. Tell them they will learn a "scientific" method of locating the decimal point in a later lesson.

1.4 The fourth basic idea is that if the result falls outside the D scale, the other index of the C scale should be used. Give the students an example such as 28.3×5.46 , and have them set the left index of C over 283 on D. They will find that they cannot move the hairline over 546 of the C scale. The result is "off the D scale." Now have the students set the right-hand index of C over 28.3 of D and finish the calculation in the usual way. Make sure they understand that by using one index or the other, the result will always be on the D scale.

Figure 10



1.5 If convenient, show the film The Slide Rule I (C and D scales). 1 reel, 16 mm., sound, 24 minutes. B & W. Castle.

1.6 Assign examples for practice, and give individual help to students who need it.

Division using the C and D scales

2. First, be sure the students understand that division is the inverse operation of multiplication. Have them set the slide rule to show $2 \times 4 = 8$. Point out that we start at 2 on the D scale, then move to 4 on the C scale, and find the answer 8 under 4 on the D scale. Have them notice that by reversing these steps we can divide 8 by 4 and get the result 2.

Figure 11



**Combined
operations using
the C and D scales**

2.1 Point out that the basic idea in dividing by slide rule is to subtract the length for the divisor from the length for the dividend and read the result on the D scale.

2.2 Second, tell the students that decimal points are to be ignored in making the settings and that the location of the decimal point is an extra step. Have them locate the point by estimating.

2.3 Third, help the students see that in division the result is never "off scale." Work out examples until most of the students feel confident that they can practice independently.

3.

To secure more practice immediately and to give students an increased sense of the convenience and power of the slide rule, many teachers will want to teach how to do "combined operations" before systematic methods of locating the decimal point are introduced.

3.1 Best results will be obtained by beginning with examples like

$$\frac{42 \times 37}{65}$$

First tell the students it is easiest to divide and multiply alternately. Thus, in the example above we should divide 42 by 65 and then multiply the result by 37. Help them to see that this means we set (i. e., move) the slide and the cursor alternately, and that we do not read intermediate results, but only the final answer.

3.2 Second, help the students work out several examples in which there are more factors; for example:

$$\frac{5.17 \times 1.25 \times 9.33}{4.3 \times 6.77}$$

Choose these so there is one more factor in the numerator than in the denominator.

3.3 Third, use an example such as:

$$\frac{3.72 \times 2.46 \times 4.31}{2.98}$$

In this example we would set the slide so 372 of C is over 298 of D, then move the hairline over 2.46 of C. The result thus far is on D under the hairline, but we still must multiply by 4.31. To do this we move the slide so 1 of the C scale is under the hairline, then move the hairline over 4.31 of C, and read the final result on D. Show the students that if desired they may write in the factor 1 in the denominator to obtain

$$\frac{3.72 \times 2.46 \times 4.31}{2.98 \times 1}$$

and this may help them to remember the settings by emphasizing the alternate "slide-hairline-slide," etc., movements.

3.4 Finally, show how an example such as $32 \times 1.65 \times 8.9$ can be done by the same principle if we write (or think)

$$\frac{32 \times 1.65 \times 8.9}{1 \times 1}$$

and again rely on the general principle discussed above.

Teaching students about approximations

Students in a modern program of mathematical instruction should learn the ideas in this section as part of their regular work. If they have learned these ideas before work in slide rule is introduced, this section may be omitted. If the students do not already have these ideas, instruction in them in connection with the slide rule is not only necessary but also is especially effective.

Helping students accept approximations to numbers

1.

Many students are reluctant to accept the approximateness inherent in much scientific work. They need to give special attention to the basic ideas involved.

1.1 Explain to the students that much scientific work and other applications of mathematics involve measurement of some kind. Remind them that measurement involves error not because of carelessness, but because of limitations of the measuring instrument. Remind them also that we can refine our instruments so our results are more precise and more accurate, but we still will have to accept small errors.

1.2 Tell the students that in slide rule work we are interested in the accuracy of our results, and that this is indicated by the number of significant digits we have in the numeral. For example, 704.05 has five significant digits. Explain that digits other than 0 are always counted as significant, but sometimes 0 is significant and sometimes it is not. A 0 between two other digits is always significant. When the number is less than 1, as for example, 0.00506, the two zeros between the decimal point and first non-zero digit are not significant. In a numeral such as 50600, the two zeros at the right of the 6 may or may not be significant. However, they are usually counted as not significant unless there is reason to believe they are significant. For example, a precise instrument might, in general, give four figures. In that case, we might know that the underlined figures in 50600 are significant but the last zero is not.

These ideas should be learned by students in their mathematics courses whether or not they study slide rules. Give students practice in counting significant figures.

1.3 Explain that we use approximations to numbers in many cases even when no measurement is involved. For example, we use 3.14 for π , and we use 1.414 for the square root of 2.

1.4 Tell students that in much scientific and engineering work results that are accurate to three significant figures are commonly accepted, at least in preliminary calculations. Remind them that this accuracy is obtainable with the slide rule--that is one reason it is so popular, and is taken everywhere as a sort of symbol for engineers and scientists.

1.5 Explain that when one number that is an approximation is multiplied or divided by another, the result is no more accurate than the least accurate of the two numbers. In a paper-and-pencil calculation of 3.82×6.29 the result, 24.0278, has six digits, but only the first three, namely 24.0, are to be taken as significant. The slide rule gives these three significant figures quickly and with no wasted work. Have them multiply out some examples using first paper and pencil, then the slide rule, and compare the two methods.

1.6 If time permits, do a few examples such as the following. Assume two measurements to the nearest inch are 26 inches and 34 inches. If new measurements precise to the nearest tenth of an inch were taken, help the students see that the results might be as large as 26.5 and 34.5, or as small as 25.5 and 33.5. Have the students multiply, by paper and pencil, 26×34 to get 884; then 26.5×34.5 to get 914.25; then 25.5×33.5 to get 854.25. Then point out the original measurements were correct to two significant figures. The result should be expressed as 880. This comes between 910 and 850, which are the rounded values of the two extreme values found above. Then show that the slide rule will also give 880.

Teaching "scientific notation"

Students in a modern program of mathematical instruction should learn the ideas in this section as part of their regular work. If they have learned these ideas before work in slide rule is introduced, this section may be omitted. If the students do not already have these ideas, instruction in them in connection with the slide rule is not only necessary but also is especially effective.

Preliminary comments

1.

The location of the decimal point in the final result has always been a serious difficulty in slide rule calculation. Many special methods have been devised, and some of these are explained in most books or manuals on the slide rule. Most of the special methods involve rules and auxiliary calculations that are different to learn, remember, and use. In fact, they are a poor substitute for the one basic or fundamental method, which uses the "scientific notation" for numbers. In this notation any real number is expressed, at least approximately, in a standard form.

The widespread use of this notation in the modern, scientific world suggests that all students ought to understand this notation. It is therefore foolish to waste time teaching most of the special "rules" for locating the decimal point.

The method of estimating the answer and locating the point is an exception, and should be taught. Actually, this method amounts to the mental use of the basic scientific notation. Unfortunately, in some examples, particularly those involving very large and very small numbers, the estimation becomes quite difficult, and for confidence in the result one must fall back on the basic method.

An effective and simple slide rule method of locating the decimal point has recently become available. It will give answers in the range 10^{-10} to 10^{10} , to one or two significant figures. However, for results more

accurate, or outside the range indicated, one must again fall back on the basic method.

For these reasons instruction should be given in high school on the use of "scientific notation." In the case of at least some of the brighter students this work can begin as early as the seventh grade. An understanding of "scientific notation" depends upon knowledge of exponents.

**Teaching
the meaning
and value
of exponential
notation**

2.
2.1 Introduce simple examples involving products of the same factor. A good example follows.
Number of parents you have is 2.
Number of grandparents is $2 \times 2 = 4$.
Number of great-grandparents is $2 \times 2 \times 2 = 8$.
In table form:

Number of Generations Back	Number of Ancestors
1	2 = 2
2	$2 \times 2 = 4$
3	$2 \times 2 \times 2 = 8$
4	$2 \times 2 \times 2 \times 2 = 16$
5	$2 \times 2 \times 2 \times 2 \times 2 = 32$
6	$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

2.2 Introduce exponent notation as a short way of writing products of the same factor; thus, write 2^5 instead of $2 \times 2 \times 2 \times 2 \times 2$.

2.3 Define terms: e. g., 2 is called the base; 5 is called the exponent; 32, or 2^5 , is called the power.

2.4 Use as an example the anecdote about the ancient monarch who wanted to reward the inventor of the game of chess and promised to give him anything he asked. The inventor said he had a very simple request. He wanted one kernel of wheat for the first square on the chessboard, two kernels for the second square, four kernels for the third square, etc. Write an expression for the number of grains in the 64th square. (Answer: 2^{63} , or $2 \times 2 \times 2 \times 2$, etc., to 63 factors)

2.5 Use an example with 10 as a base. Have the class prepare a table as follows:

Exponent	Factors Written Out	Power in Exponent Notation	Name of Number
1	10	10^1	"ten"
2	10×10	10^2	"one hundred"
3	$10 \times 10 \times 10$	10^3	"one thousand"
4	$10 \times 10 \times 10 \times 10$	10^4	"ten thousand"
5	$10 \times 10 \times 10 \times 10 \times 10$	10^5	"one hundred thousand"
6	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	10^6	"one million"

2.6 Summarize to bring out: (a) how exponents make it easy to write powers of ten; (b) the exponent tells the number of times that 10

is used as a factor; (c) for powers of ten the exponent is found by counting the number of zeros in the repeated factor.

2.7 Assign examples for practice. The following are typical of what you may use.

Write in exponential notation:

- (a) $4 \times 4 \times 4$ (c) $6 \times 6 \times 6 \times 6 \times 6 \times 6$
(b) $3 \times 3 \times 3$ (d) $10 \times 10 \times 10 \times 10 \times 10 \times 10$

Write the following in the usual way:

- (a) 5^8 (c) 3^5 (e) 10^4
(b) 10^8 (d) 4^6 (f) 6^4

**Teaching how
to write numbers
in "standard form"
or "scientific
notation"**

3.

3.1 Tell the students that every number, or at least an approximation to it, can be written in one "standard form." In this form the decimal point in the numeral is written to the right of the first digit, and then a factor which is a power of 10 is written. Work out examples such as the following:

- (a) $254 = 2.54 \times 100 = 2.54 \times 10^2$
(b) $683,000 = 6.83 \times 100,000 = 6.83 \times 10^5$

3.2 Explain the advantages of this way of writing the numerals for numbers. Among these advantages are the following:

3.21 In many cases it is shorter. Thus, $2.3 \times 10^{13} = 23,000,000,000,000$. In scientific work the numbers sometimes are very large or very small.

3.22 When numbers are expressed in this form and then multiplied or divided, etc., the location of the decimal point in the answer is simplified.

3.23 This form makes clear just how accurate an approximation one is dealing with. Thus, we say that 4.82 is "accurate to three significant figures." If we see the numeral 482,000, we cannot be sure we have six significant figures. It is more likely that we still have only three significant figures. To show this we can write 4.82×10^5 instead of 482,000. If we actually have six significant figures, we should write 4.82000×10^5 . If we have four significant figures, we should write 4.820×10^5 . A recent estimate of the age of the solar system is 4,950,000,000 or 4.95×10^9 years. This approximation is certainly not accurate beyond three significant figures. Help the students see that this way of showing the accuracy of the result is important in careful scientific work.

3.3 Explain how to write numerals in "scientific notation." The easiest method is as follows:

- (a) Write the significant digits, a "times" sign (\times), and a 10.
(b) Place a decimal point at the right of the first (counted from the left) non-zero digit.
(c) Start at the right of first non-zero digit in the original numeral and count the "places"--that is, the digits and zeros passed over--in reaching the decimal point. The result of the count is

the absolute, or "numerical," value of the exponent. If the count to the original decimal point moves toward the right, the exponent is positive (+). If the count moves toward the left, the exponent is negative (-). Write in the proper symbol for the exponent of 10.

For example, take 5,790,000. First, write 5.79×10 . Next, start between the 5 and the 7 and count (toward the right) the six "places." Write the 6 to indicate the power of 10.

For 0.000283, first write 2.83×10 . Now start between the 2 and the 8 and count (toward the left) the four "places." The final result is, then, 2.83×10^{-4} .

3.4 Show students how to change a numeral from standard form to ordinary form by reversing the above process. Thus, to express 4.68×10^5 in ordinary form write 468 and then supply zero symbols until there are 5 places to the right of the 4; the result is 468,000.

To express 8.93×10^{-3} in ordinary form write 893 and then supply zero symbols to the left of the 8 until there are 3 places, including the 8; the result is 0.00893.

3.5 Provide an adequate supply of examples for practice.

**Teaching how to use
"scientific notation"
to locate
the decimal point
in multiplication**

4.

The use of "scientific notation" to locate the decimal point requires the ability to multiply and divide powers of ten. If students already know the "laws of exponents" and how to calculate with the integers (that is, positive and negative numbers and zero), the use of this knowledge in locating the decimal points should be merely a highly valid application. If they do not have this knowledge, they must acquire it either formally or informally. We will assume here that the students do not have this knowledge and that it is to be taught informally. If the teacher wishes, and time permits, a more formal approach of the type found in algebra textbooks may be used. This will, however, be somewhat of a digression if injected in a unit on the slide rule.

4.1 Begin by giving examples such as $17,400,000 \times 248,000$. Point out that if done in the usual way a good many zeros must be written. Have the students express the product in scientific notation

$$1.74 \times 10^7 \times 2.48 \times 10^5.$$

Remind them that in multiplication the result does not depend on the order in which the factors are taken. Have them multiply 1.74×2.48 on the slide rule to get 4.32. Now point out that in 10^7 and 10^5 together they will have 10 as a factor $7 + 5$ or 12 times. Thus, the result is 4.32×10^{12} . Have them write this out in ordinary notation by counting 12 places to the right from the 4.

4.2 Next, use examples such as $17,400,000 \times 0.0000248$. This example may be written in the form $1.74 \times 10^7 \times 2.48 \times 10^{-5}$. Point out that if they write out the answer in ordinary form, they could do it in two steps. First, they would count 7 places to the right from the 4 in 4.32. Then, because of the 10^{-5} , they should count 5 places

back to the left. They should see that this will bring them just two places to the right of the 4, so the result is 4.32×10^2 or 432.

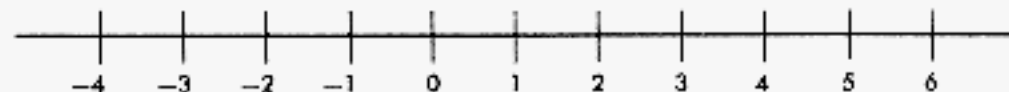
Do several examples this way until the students begin to see that a shorter method is to subtract 5 from 7; that is, to find $7 - 5 = 2$, and then count just two places to the right.

4.3 Help the students to generalize the procedure. Give examples such as:

$$\begin{aligned} &17,400,000 \times 0.000,000,002,48 \\ \text{or} &1.74 \times 10^7 \times 2.48 \times 10^{-9}. \end{aligned}$$

They should see that the counting process will take them two places to the left of the decimal point in 4.32; that is, the result should be 4.32×10^{-2} or 0.0432. If necessary, use a "number line" labeled as in the figure below to help them visualize this. Show that if they

Figure 12



start at 0 and count 7 places to the right, then count back 9 places, they will end up at -2, or 2 places to the left of 0.

4.4 Continue this informal use of "signed numbers" by using examples, such as:

$$\begin{aligned} &0.000,174 \times 0.002,48 \\ \text{or} &1.74 \times 10^{-4} \times 2.48 \times 10^{-3} \end{aligned}$$

The students should see that the count now will take them 7 places to the left of the decimal point in the 4.32. The result is 4.32×10^{-7} , which is 0.000,000,432.

4.5 Show students the meaning of situations such as $4.32 \times 10^3 \times 10^{-3}$. Point out that this indicates that one should count 3 places to the right, then count back 3 places to the left. Thus, one arrives at the place where he started. Observe that $3 - 3 = 0$, so the example can be written 4.32×10^0 . The 0 can be interpreted as calling for no counting either way. In effect, one "stays put," or "stands still." The result, then, is 4.32.

Tell the students that mathematicians have agreed to interpret 10^0 as 1, since the product of 1 and any factor is just that factor.

4.6 Finally, show the students how to adjust the powers of ten in cases where the product of the non-exponential factors is 10 or more. For example, consider:

$$6.28 \times 10^3 \times 4.32 \times 10^2 = 6.28 \times 4.32 \times 10^5.$$

Here $6.28 \times 4.32 = 27.1$, or 2.71×10^1 . Because of this "extra" factor, the result is 2.71×10^6 .

4.7 Give adequate practice. Remember that this ability is important in its own right, and should be learned even if students are not expecting to use it in slide rule work.

**Teaching students
to use
"scientific notation"
to locate
the decimal point
in division**

5.

5.1 Begin by giving an example such as:

$$4,320,000,000,000 \div 248,000.$$

Have students write this in the form

$$\frac{4.32 \times 10^{12}}{2.48 \times 10^5} = \frac{1.74 \times 10^{12}}{10^5},$$

and divide by using the slide rule. The students must next learn to get a single power of ten in the numerator. If they know how to divide exponentials, this is no problem. We assume they do not, and continue with less formal but meaningful methods.

5.2 Help the students recall that in a division example both "terms" (i. e., dividend and divisor) may be multiplied or divided by the same number without changing the quotient. Use examples such as the following:

$$\frac{12}{3} = 4; \frac{12 \times 10}{3 \times 10} = \frac{120}{30} = 4; \frac{12 \div 10}{3 \div 10} = \frac{1.2}{.3} = 4.$$

$$\frac{48}{6} = 8; \frac{48 \times 100}{6 \times 100} = 8; \frac{48 \div 100}{6 \div 100} = 8.$$

5.3 Returning to the example (see 5.1)

$$\frac{1.74 \times 10^{12}}{10^5},$$

bring out that we would like the divisor to be 1, and can accomplish this by multiplying both dividend and divisor by 10^{-5} ; in other words, we "count back" 5 places in each. The result is 1.74×10^7 . Intermediate steps, namely

$$\frac{1.74 \times 10^{12} \times 10^{-5}}{10^5 \times 10^{-5}} = \frac{1.74 \times 10^7}{10^0} = \frac{1.74 \times 10^7}{1},$$

can be written, but eventually and for speedy work this is usually unnecessary. Help the students see that this procedure of making the divisor 1 is always possible.

**Teaching
decimal point
location in
combined
operations**

6.

Use a few examples to show students how to handle more complicated situations, such as the following:

$$\frac{3.65 \times 10^3 \times 4.81 \times 10^{-7}}{3.47 \times 10^{-4} \times 2.61 \times 10^2}$$

By estimation, we see that the non-exponential factor will reduce to about $\frac{3}{4}$ or 0.75, and the exponentials boil down to 10^{-2} . By slide rule, we then get $0.794 \times 10^{-2} = 7.94 \times 10^{-3} = 0.00794$.

Using the "Powers-of-Ten" slide rule as a learning aid in teaching exponential notation and decimal point location

This section may be regarded in several ways.

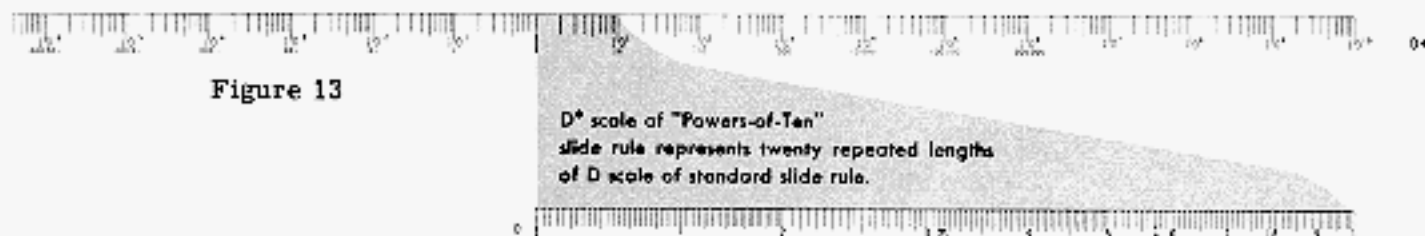
It provides an alternative approach to "scientific notation" through the use of a special learning aid. At the same time, it introduces the soundest approach to a purely mechanical way of locating the decimal point that has yet been devised.

If "Powers-of-Ten" scales cannot be made available, or if time is very limited, this section may be omitted.

If "Powers-of-Ten" scales are available, they may be used to introduce the slide rule and solve problems. This approach avoids the usual difficulties of decimal point location at the outset.

However, accuracy of at most two significant figures is obtainable.

The "Powers-of-Ten" slide rule has non-uniform logarithmic scales of the type shown in Figure 8 on page 11. The range is 10^{-10} to 10^{10} . The primary graduations are uniformly spaced, and represent powers of ten. The secondary graduations are non-uniformly spaced. They correspond to the primary graduations on an ordinary C or D scale. Thus, the D* and C* scales represent twenty repeated lengths of the "ordinary" D or C scale as seen on the standard slide rule, but they have been greatly reduced. Looked at another way, the D scale of the standard slide rule may be regarded as one small section of the D* scale which has been greatly enlarged.



The "Powers-of-Ten" slide rule may be used to solve problems in a simple way that takes account of the decimal point at all times. It may also be used as a teaching aid to help students learn how to use exponential notation and the standard form of numerals.

Multiplying powers of ten

1. Begin by showing students how to multiply powers of ten. For example, show $10^2 \times 10^3$ as

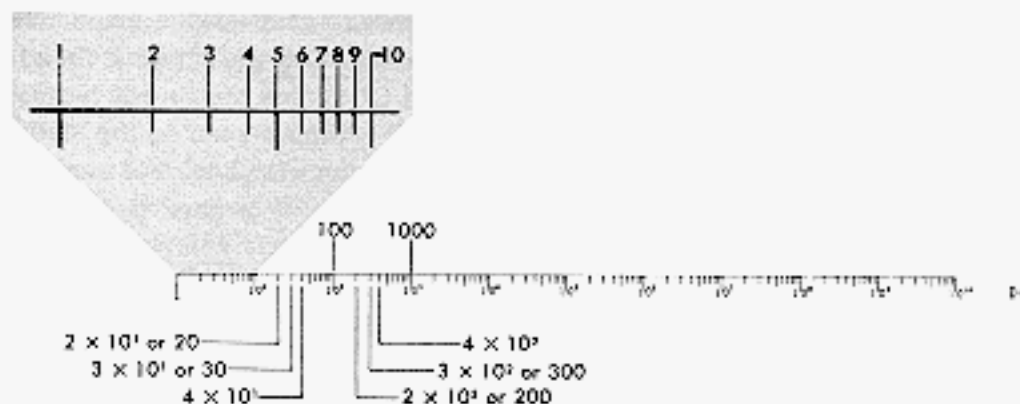
$$\underbrace{10 \times 10}_{10^2} \times \underbrace{10 \times 10 \times 10}_{10^3} \text{ or } 10^2 \times 10^3 = 10^5.$$

Work through several other examples, but do not strive for "mastery" at this point.

Introducing the "Powers-of-Ten" slide rule

2. Introduce the "Powers-of-Ten" slide rule as a mechanical device for multiplying and dividing powers of ten.
 - 2.1 Call attention to numerals on the C* and D* scales to the right of 1, i.e., $10^1, 10^2, 10^3$, etc., 10, 100, 1,000.
 - 2.2 To show $10^2 \times 10^3 = 10^5$, have students proceed as follows: Place 1 of C* over 10^2 on D*. Under 10^3 of C* read 10^5 on D*. Work out similar examples with the slide rule, and check a few by writing out factors of 10 as was done above.
 - 2.3 Tell students that by learning to read the scales they can do any multiplication example easily, and the location of the decimal point in the answer will be known immediately.
 - 2.4 Teach students how to read the scales for settings between primary subdivisions. Use a large chart on the board, similar to the chart below.

Figure 14



Have students move the hairline of the "Powers-of-Ten" slide rule to the right from graduation to graduation and read values as shown above.

Multiplication by the "Powers-of-Ten" slide rule

3.
 - 3.1 Begin with a few simple examples such as $20 \times 300 = 6000$. Place 1 of C* over 20 on D*. Under 300 of C* read 6000 on D*.

Also write out these factors and the product in exponential or "powers of ten" form as below:

$$\begin{array}{c} \overbrace{2 \times 10^1}^{20} \times \overbrace{3 \times 10^2}^{300} \\ \downarrow \quad \downarrow \\ 2 \times 3 \times 10 \times 10^2 \end{array}$$

or by changing the order of the factors, get 6×10^3 , which is 6000.

3.2 Next introduce examples such as 24×362 .

Place 1 of C* over 24 on D*. (Note: set 24 roughly between 20 and 30.)

Under 362 of C* read 8×10^3 or about 8000.

Write this out in scientific notation as:

$$\begin{array}{l} \text{Change order} \quad 2.4 \times 10 \times 3.62 \times 10^2 \\ \text{Multiply out} \quad 2.4 \times 3.62 \times 10 \times 10^2 \\ \quad \quad \quad 8.388 \times 10^3, \text{ or } 8388. \end{array}$$

3.3 Explain how the "Powers-of-Ten" slide rule gives an approximation (8000+) which serves as a check--especially for the decimal point, which is automatically placed.

Introducing negative exponents

4.

4.1 Call attention to the portion of the D* scale of the "Powers-of-Ten" slide rule to the left of 1. Note powers are 10^{-1} , 10^{-2} , 10^{-3} , etc. Help students build a chart on the board as follows:

Numbers	Factors	In scientific notation
1,000,000	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	1×10^6
100,000	$10 \times 10 \times 10 \times 10 \times 10$	1×10^5
10,000	$10 \times 10 \times 10 \times 10$	1×10^4
1,000	$10 \times 10 \times 10$	1×10^3
100	10×10	1×10^2
10	10	1×10^1
1	1	1×10^0
.1	$\frac{1}{10}$	1×10^{-1}
.01	$\frac{1}{10 \times 10}$	1×10^{-2}
.001	$\frac{1}{10 \times 10 \times 10}$	1×10^{-3}
.0001	$\frac{1}{10 \times 10 \times 10 \times 10}$	1×10^{-4}
.000,01	$\frac{1}{10 \times 10 \times 10 \times 10 \times 10}$	1×10^{-5}
.000,001	$\frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10}$	1×10^{-6}

Help students generalize that a negative exponent indicates division by a power of 10.

4.2 Extend multiplication to include numbers which require negative exponents when expressed in scientific notation. Use examples such as $10^3 \times 10^{-2}$, as follows: Place 1 on C* over 10^3 on D*. Under 10^{-2} of C* read 10^1 on D*.

4.3 Show how to find $10^3 \times 10^{-2}$ in two ways:

$$(a) \quad 10 \times 10 \times 10 \times \frac{1}{10 \times 10} = 10,$$

$$(b) \quad \begin{array}{l} 10^3 = 1000 \\ \frac{10^{-2}}{10^1} = \frac{.01}{10.00}. \end{array}$$

4.4 Practice multiplication on the "Powers-of-Ten" slide rule, using examples, such as:

$$(a) \quad 10^5 \times 10^{-3} = 10^2$$

$$(c) \quad 10^4 \times 10^{-7} = 10^{-3}$$

$$(b) \quad 10^{-3} \times 10^8 = 10^5$$

$$(d) \quad 10^{-3} \times 10^{-2} = 10^{-5}$$

4.5 Extend to numbers in scientific notation, for example:

$$\begin{aligned} 4 \times 10^3 \times 2 \times 10^{-5} &= 8 \times 10^3 \times 10^{-5} \\ &= 8 \times 10^{-2} \\ &= .08 \end{aligned}$$

4.6 Explain how to write numbers in scientific notation easily when the exponent is negative. For example, consider

$$0.000623 = 6.23 \times 10^{-4}.$$

As before, start at the right of the first significant digit (here 6), and count places to the actual position of the point (here 4 places to the left). If the count is to the left, the exponent is negative.

4.7 Solve problems, such as:

$$.000623 \times 847.$$

This can be written $6.23 \times 10^{-4} \times 8.47 \times 10^2$,

or $6.23 \times 8.47 \times 10^{-4} \times 10^2$,

5.3×10^{-1} , or 0.5, approximately.

(Place 1 on C* over 6×10^{-4} on D*.

Under 8×10^2 on C* read 5.3×10^{-1} , or 0.5 on D*.)

Introducing division using exponents

5.

5.1 Begin with simple examples, such as:

$$6000 \div 20, \text{ or}$$

$$6 \times 10^3 \div 2 \times 10.$$

Write this in the form $\frac{6 \times 10^3}{2 \times 10} = \frac{6 \times 10 \times 10 \times 10}{2 \times 10}$.

Divide to get $3 \times 10 \times 10$ or $3 \times 10^2 = 300$.

5.2 Introduce division by "Powers-of-Ten" slide rule. Use simple examples, such as:

$$(a) \quad 10^3 \div 10^1$$

Over 10^3 on D* place 10^1 on C*.

Read 10^2 on D* under 1 on C*.

$$(b) \quad 10^2 \div 10^{-4}$$

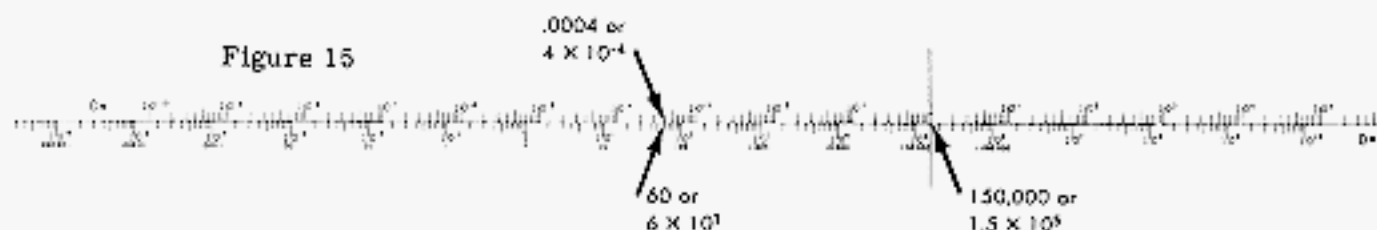
Over 10^2 on D* place 10^{-4} on C*.

Read 10^6 on D* under 1 on C*.

5.3 Remind the students of the inverse relation of division and

multiplication, showing how division is just the opposite of multiplication as far as slide rule operation goes.

5.4 Next consider more representative examples such as $62.3 \div 0.00042$. Set 4×10^{-4} on C^* over 62 (or $6. \times 10$) on D^* . Under 1 on C^* read 1.5×10^5 (or 150,000) on D^* .



5.5 Summarize the procedure and help students understand that the procedure in setting with C^* and D^* is exactly the same as it is with the C and D scales. However, the setting is made in the portion of the C^* and D^* scales that corresponds to the actual magnitude of the number. Thus, numbers between 10,000 and 100,000 are set in the section between 10^4 and 10^5 . Numbers between 0.0001 and 0.001, such as 0.00068, are set in the portion between 10^{-4} and 10^{-3} .

With care, settings to two significant digits may be approximately made. There is some loss of accuracy, but the decimal point in the answer is obtained along with its first one or two significant figures.

5.6 Have students work problems that involve combined operations.

Teaching students to use the A and B scales

This section may be deferred until after the CI scale has been introduced. One authority suggests teaching the use of the A and B scales even before multiplication with the C and D scales has been taught. The argument for this is that finding square roots by ordinary arithmetic is difficult, but by slide rule is easy. Thus, an advantage of the slide rule can be shown effectively at the start. On the other hand, the A and B scales are graduated differently from the C and D scales, and other more serious difficulties in reading scales are also met.

The students will undoubtedly have noticed the A and B scales and may have asked about their use. Tell them these scales are used to find squares and square roots.

Helping students become acquainted with the A and B scales

1. 1.1 Have the students set the hairline over 2 on D and read its square, 4, on the A scale. Then move the hairline to 3 and read 9 on A. Generalize to the rule that the square of any number set on D may be read on A, and conversely.

Have the students set the hairline over 2 on A and read its square root, 1.41, on D. Also, move the hairline to 3 on A and read $\sqrt{3}$, or 1.732, on D. Point out that finding square roots is easy when a slide rule is used.

1.2 Help the students see that the A scale is similar to the D scale but is reduced in length so that there is an index at the midpoint of the scale. Thus, if the left-hand index is read as 1, this middle index may be read as 10, and the right-hand index will then be read as 100. If the left index is read as 100, the middle index would be read as 1000, and the right index as 10,000.

The shorter sections make it necessary for the graduation system to be altered. Teach students to read the A and B scales using methods similar to those suggested earlier for the C and D scales.

1.3 Tell the students that the A and B scales are useful in combined operations. Use simple examples such as $2 \cdot 9$ and $3 \cdot 2$ to illustrate the principle. To find $2 \cdot 9$ have the students set 1 of C over 2 on D, then move the hairline over 9 on B, and read the answer 6 on D. Show how

this setting of the slide gives 2 times the square root of any number set on B.

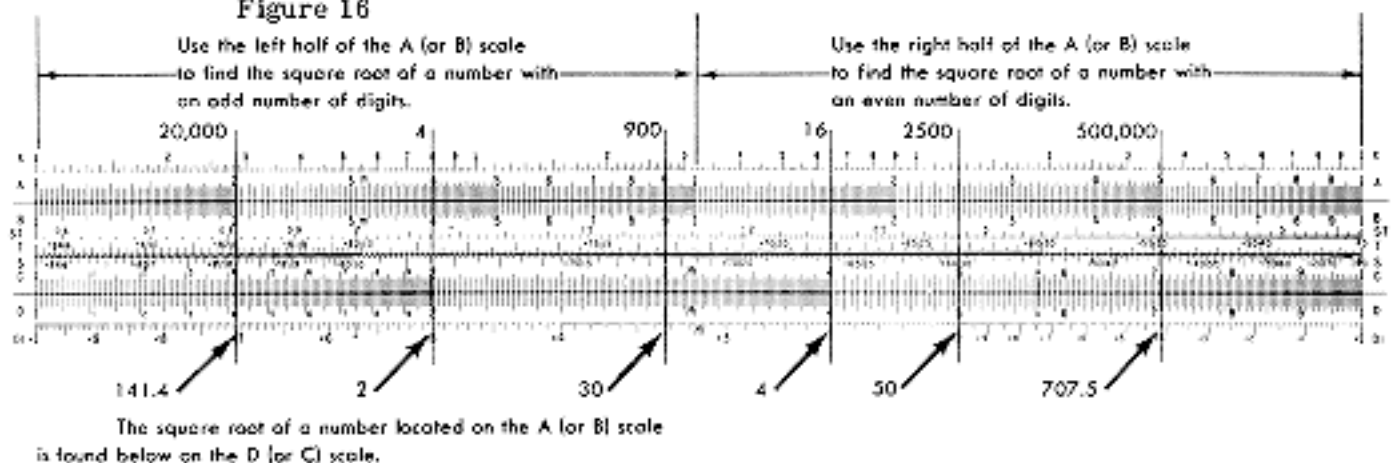
Next, use simple examples such as 2×3^2 and 4×7^2 to show how to do examples of the form $a \times b^2$. Have the students set 1 of B under 2 of A, then move the hairline to 3 on C. They can then read the answer 18 on A. Generalize the procedure so they see that when an index of D is set under a on the A scale, and the hairline is moved to b on the C scale, the product ab^2 can be read on the A scale.

**Helping students
make decisions
relating to decimal
point location**

2.

Once the general ideas are established by methods such as those suggested above, special attention must be given to reading the scales and to decimal point location, especially in finding square roots. Two methods are suggested. First, help students see that if a numeral has an odd number of digits (as, for example, 4, or 900, or 20,000), the setting to find the square root should be made on the left-hand section of the A or B scale. If the numeral has an even number of digits (as, for example, 16, or 2500, or 500,000), the setting to find the square root should be made on the right-hand section of the A or B scale.

Figure 16



2.1 The easiest way to approach this is to think of the numeral in "periods" of 2 digits each, counting both ways from the decimal point. For example, for the numeral 3820000, the separation 3'83'00'00 shows four periods. The one on the left has only one digit, and we therefore have an odd number of digits in all. Hence, the left portion of A would be used. There will be as many "places" to the left of the decimal point in the numeral for the square root as there are periods in the given numeral. In the case of the example, then, the square root is 1954.

On the other hand, in the case of fractions we can count the number of zeros between the decimal point and the first significant digit. Thus, 0.00'02'5 has 3 such zeros, and so to find the square root 25 would be set on the left section of the A scale. The square root is 0.01581, where one 0 must be inserted at the left of the first significant digit to locate the decimal point. This one 0 corresponds to the period '.00' in the given numeral.

However, the numeral 0.00'00'25 has 4 zeros, so the right-hand section of the A scale is used. The square root is 0.005, in which two placeholder symbols '00' are used, one for each of the periods in the original numeral:

$$0.000025 = 0.005.$$

2.2 A more fundamental and more difficult approach uses the powers-of-ten notation. After the numeral is written in standard form, it must sometimes be adjusted so that the exponent is an even number. To find the square root of 3,820,000 we write 3.82×10^6 , and we can find the square root of 3.82 by using the left section of the A scale. The result, 1.954, is then multiplied by 10^3 , in which the exponent 3 is $6 \div 2$. Thus, $3,820,000 = 1.954 \times 10^3$ or 1954. In general, the exponent must be divisible by 2.

Now to find $\sqrt{382,000}$ we first write 3.82×10^5 . However, this must be rewritten in the form 38.2×10^4 . Now 38.2 is found by using the right section of the A scale. Thus, one obtains 6.18×10^2 , or 618, as the result. Similarly, to find $\sqrt{0.00025}$ we write 2.5×10^{-4} , and (using the left section) obtain 1.581×10^{-2} , or 0.01581. But to find $\sqrt{0.000025}$ one must change 2.5×10^{-5} to 25×10^{-6} , and obtain (using the right-hand section of the A scale) 5×10^{-3} , or 0.005.

2.3 The readjustment of the standard form described in ¶2.2 requires thought and care. Students who are sufficiently advanced may not find the method very difficult to understand and learn, but many junior high school students will find it far from easy. For them, and in fact in most practical work, the data should normally be such that the determination of which section of the scale to use and the location of the decimal point in the answer may usually be made by "common sense" estimates. On the other hand, if sufficient time is available, instruction in the adjustment of the "scientific notation" can readily be justified for students who are at all likely to go on to scientific or technological work of any kind.

The rather mechanical procedure will be better understood if the students examine a table of squares and observe how the number of digits in the square varies with the number of digits in the given number. Thus, "one-digit numbers" have squares of either one or two digits: $3^2 = 9$, but $4^2 = 16$. In going from a given number to its square root, then, the "period" device is a sort of mechanical means of indicating how many digits (or zeros to the left of the first significant digit) there will be in the square root.

2.4 If convenient, show the film The Slide Rule II (A and B Scales). 1 reel, 16 mm., sound, 21 minutes. B & W, Castle.

2.5* If "Powers-of-Ten" scales A* and B* are available, show the students how they can be read for finding squares and square roots. With these scales none of the usual difficulties in deciding whether the left-hand or right-hand sections of the A and B scales are met. The location of the decimal point is given by the scales. The range of A* and B* is from 10^{-20} to 10^{20} .

Teaching students to use the K scale

In general, the suggestions given above for the work in the A and B scales may be adapted for teaching the K scale. They will be outlined very briefly here.

1. Help the students learn that the K scale is used for cubes and cube roots. Have them note it has 3 sections, and if the left index is read as 1, the others represent 10, then 100, and finally 1000.

2. Help the students learn to read the scale.

3. Help students solve examples, such as $3\sqrt[3]{8}$. Some slide rules place the K scale on the slide. If so, it is read against C. Others place the K scale on the body of the rule. If so, it is read against D. Most slide rules have either one or the other of these arrangements, and this limits the flexibility of operation. One model has a K scale on both slide and body, and this model has flexibility in combined operations with cubes and cube roots that is comparable to the situation for squares and square roots.

4. Teach some method of decimal point location very carefully. Periods of three digits (or zeros) can be counted off in a fashion similar to that used for square roots.

An adjustment of the standard form can be made so that the exponent is divisible by 3. Thus, to find $\sqrt[3]{45700}$ we write $4.57 \times 10^4 = 45.7 \times 10^3$. Use the middle section of K to get 3.58×10 , or 35.8 for the result.

5. Throughout, emphasize that although the slide rule method of finding cubes and cube roots may seem a little difficult at first, it is very much easier to learn, remember, and use than the traditional arithmetic methods.

Teaching students to use the CI scale

The material discussed in this section could be introduced much earlier than its appearance in this outline might suggest.

However, beginners can easily become confused in using the CI scale.

For this reason it is best to defer teaching it until after the students have gained some confidence in using the more basic scales.

In this section the paragraphs indicated by a star (e. g., 3*) apply only after Section VI has been taught.

The students will undoubtedly have noticed the CI scale and may have asked what it was used for. Tell them they will now learn what it is used for and why it is very convenient to have on a slide rule.

1. First, make sure the students see that the CI scale is like the ordinary C scale except that the graduations and numerals are read from right to left, instead of in the other direction. Have them move the hairline along and read some of the graduations.

2. Next, remind the students of the meaning of the term reciprocal. If they are not familiar with the term, tell them that two numbers whose product is 1 are called reciprocals; for example, 2 and $1/2$, or 2 and 0.5, are reciprocals. Call for other easy examples.

3. Have the students move the hairline over 2 on C and notice it is automatically over .5 on CI. When it is over 4 on C, it is over 0.25 on CI. Also have them read from CI to C. Be sure they understand that the reciprocal of any number set on C may be read on CI, and conversely. Have them practice finding the reciprocal of a given number.

3. * If "Powers-of-Ten" slide rules are available, have the students notice that C* and CI* give reciprocals with decimal points located. Thus, if the hairline is placed over the graduation for 500 on C*, it will also be over 2×10^{-3} , or 0.002, on CI*.

4. Next, give an example such as $2 \div 8$. Have the students notice that when 8 on C is set over 2 on D, the slide "sticks out" rather far

to the left. Show them that this can be avoided by using the CI scale. That is, have them notice that $2 \div 8 = 0.25$ is the same as $2 \times \left(\frac{1}{8}\right) = 0.25$. In general, help them see that a divided by b can be replaced by a multiplied by $\frac{1}{b}$; that is, by a multiplied by the reciprocal of b . Have them set 1 of C over 2 on D, then move the hairline to 8 on CI and read 0.25 on D. Call attention to the fact that usually the slide does not need to be moved so far to do an example this way. Give an ample set of examples for practice in the use of the CI scale.

5. Have the students observe that 2×8 can be replaced by $2 \div \left(\frac{1}{8}\right)$, and, by use of similar examples, have them make the generalization that a multiplied by b can be replaced by a divided by the reciprocal of b . Have them set 1 of C over 2 of D, and notice that 8 of C falls "outside the rule." Show that it is not necessary to move the slide end for end. Instead, set 8 on CI over 2 on D and under 1 of CI read 16 on D. Thus multiplication by 8 has really been replaced by a division by $1/8$.

6. Finally, show how combined operations can be simplified by re-writing them using reciprocals. For example, consider

$$\frac{151}{3.26 \times 2.43} = \frac{151 \times \left(\frac{1}{2.43}\right)}{3.26}$$

Work this out on the slide rule by two methods.

First method:

- Set 326 on C over 151 on D.
- Move hairline over 1 of C.
- Move slide so that 243 on C is under the hairline.
- Under 1 of C read 19.05 on D.

Second method:

- Set 326 on C over 151 on D.
- Move hairline over 243 on CI.
- Read 19.05 under hairline on D.

Have the students notice that in the second method using CI only one setting of the slide was needed. Point out that this method is not only easier and shorter, but also reduces the possibility of error in re-setting the slide.

6. * Show students how to use CI* in combined operations.

Teaching students the meaning of logarithms, how to find logarithms, and how the L, C, and D scales are made

In this section it is assumed that the students have not studied logarithms before instruction on the slide rule is begun.

In this case, the slide rule itself makes an effective learning aid for the introduction of logarithms.

The ordinary slide rule is constructed by making use of logarithms, and its rules of operation are based upon corresponding principles in the theory of logarithms. The slide rule can be used as a learning aid to teach students some ideas about logarithms. Or, more fundamentally, if students understand logarithms, this understanding contributes to a basic understanding of the slide rule.

Helping students understand what a logarithm is

1. 1.1 Tell students that the term logarithm is another name for exponent. Remind them that the exponents they have thus far considered have usually been integers--that is, positive or negative "whole numbers," and zero. Ask them what they think a symbol such as $3^{\sqrt{2}}$ or $10^{0.301}$ might represent. After discussion bring out that in cases such as these, where the set of numbers used as exponents has been extended, the term logarithm is frequently used.

1.2 Tell the students to assume the left index represents $1 = 10^0$. Then, the right index represents $10 = 10^1$. Ask them, "What does the graduation labeled 2 represent, expressed as a power of 10?" In other words, this question calls for the solution of the equation $2 = 10^x$. Ask, "What number must we use as the exponent of 10 to get 2?" Bring out that a number between 0 and 1 is being sought. Explain that mathematicians have a method of calculating this number, and that it is

0.30103 carried to six significant digits. In similar fashion, the exponent of 10 which produces 3 as the power is 0.477121. We say, "The logarithm of 3 to base 10 is 0.477121." This is abbreviated by writing $\log 3 = 0.477121$. Other logarithms are given in the table. In general, if

$$10^{\underline{a}} = \underline{b}, \text{ then } \log \underline{b} = \underline{a}.$$

Short Table of Logarithms

Number	1	2	3	4	5	6	7	8	9	10
Power of Ten	10^0	$10^{0.301}$	$10^{0.477}$	$10^{0.602}$	$10^{0.699}$	$10^{0.778}$	$10^{0.845}$	$10^{0.903}$	$10^{0.954}$	10^1
Exponent or Logarithm	0	0.301	0.477	0.602	0.699	0.778	0.845	0.903	0.954	1

**Helping students
find logarithms
by slide rule**

2.

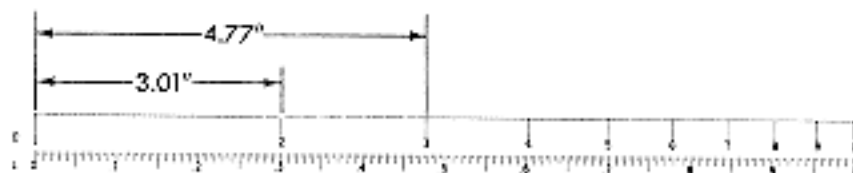
Call attention to the L scale on the slide rule. On some models it is placed on the slide and is read with C. On other models it is placed on the body and read with D. Point out that it is a uniform scale and can be used to measure distance. Point out also that it is graduated in tenths, and that these marks are labeled .1, .2, .3, . . . , and the decimal points are shown. It is also graduated in tenths of tenths, or hundredths, but these marks are not labeled. Finally, between these graduations are others which enable us to read the L scale to thousandths. Help the students locate a few graduations such as 0.372, 0.698, etc.

**Helping students
understand
how the C and D
scales are made**

3.

3.1 Remind the students that $\log 2 = .301$. Have them place the hairline over .301 on L and observe that on the C (or D) scale the corresponding mark is labeled 2. Similarly, opposite .477 on L is the mark 3 on C (or D). Explain that this is the way a C scale is made. That is, for example, one can start by measuring .301 units on L, then putting a graduation at that point on C and labeling this with 2. Similar measurements on L using values from the table of logarithms can be used to locate the other primary graduations on C (or D).

Figure 17



3.2 Bring out clearly that if the logarithm is given and its graduation located on L, the numeral for the corresponding number can be read from C (or D). Conversely, if a number is given, its logarithm can be found. The graduation for the number is located on C (or D), and the corresponding mark on L is read to determine the logarithm. If time permits, have the students make a C scale on paper.

3.3 Next, explain that the secondary graduations on C (or D) are located by similar methods. For example, the logarithm of 1.5 is calculated to be 0.176. The graduation for 1.5 on C (or D) is placed opposite the graduation for 0.176 on L. This process could be continued indefinitely, but the graduations would soon become very close together.

3.4 Finally, remind the students that when a numeral is in standard form, it has one factor which is a number between 1 and 10. The graduation for this factor can be located on the C (or D) scale. Its logarithm is always a decimal fraction.

The other factor is a power of 10 which has an integral exponent. For example, have the students consider 1.5×10^y . Have them write 1.5 in exponential form to get:

$$10^{0.176} \times 10^y = 10^{0.176 + y} \text{ or } 10^{y + 0.176}.$$

Now have the students build a table of a whole set of numbers which have 15 as the significant digits; thus

1,500,000	= 1.5×10^6	= $10^{0.176} \times 10^6$	= $10^{6.176}$
150,000	= 1.5×10^5	= $10^{0.176} \times 10^5$	= $10^{5.176}$
15,000	= 1.5×10^4	= $10^{0.176} \times 10^4$	= $10^{4.176}$
1,500	= 1.5×10^3	= $10^{0.176} \times 10^3$	= $10^{3.176}$
150	= 1.5×10^2	= $10^{0.176} \times 10^2$	= $10^{2.176}$
15	= 1.5×10^1	= $10^{0.176} \times 10^1$	= $10^{1.176}$
1.5	= 1.5×10^0	= $10^{0.176} \times 10^0$	= $10^{0.176}$
0.15	= 1.5×10^{-1}	= $10^{0.176} \times 10^{-1}$	= $10^{0.176-1}$
0.015	= 1.5×10^{-2}	= $10^{0.176} \times 10^{-2}$	= $10^{0.176-2}$
0.0015	= 1.5×10^{-3}	= $10^{0.176} \times 10^{-3}$	= $10^{0.176-3}$
0.00015	= 1.5×10^{-4}	= $10^{0.176} \times 10^{-4}$	= $10^{0.176-4}$

From this, help them see that changing the position of the decimal point changes only the integral part of the logarithm or exponent. Because of this, the L scale can be used to obtain the decimal fraction part of the logarithm of any number, ignoring the position of the point in the numeral for the number itself. Then, the integral part, which is usually called the characteristic, is found by counting in the same way as is done when one writes the numeral in standard form. The characteristic is the count. The decimal fraction, or mantissa, is obtained from a table, or from the L scale.

3.4* If "Powers-of-Ten" slide rules are available, the students should observe that the L* scale has range -10 to 10, whereas the L scale has range only from 0 to 1. When a number is set on D*, its logarithm including the characteristic is read on the L* scale. Thus, under 2×10^4 on D* one reads 4.3 on L*. Conversely, if one has the logarithm and locates its graduation on L*, the numeral for the corresponding number is readable on D* in standard form. Thus, if the logarithm is 6.477, one finds 3×10^6 , or 3,000,000, on D*.

Teaching the use of the S, T, and ST scales

It is assumed that most high school classes will not have time enough available for instruction on slide rule methods to enable the students to acquire any real competence in using the S, T, and ST scales. Hence, only a brief outline is presented here. In case adequate time is available, instruction on the use of the slide rule in trigonometric calculations can be greatly extended. In such cases the instructor will usually have many ideas of his own as to how the material may best be presented.

The trigonometric scales S, T, and especially ST, are not provided on some slide rule models. They are, however, very valuable for many types of problems, and, if time permits, students should be given some help in learning to use them. Needless to say, this is not desirable unless the students are already familiar with the trigonometric functions named "sine" and "tangent."

Helping students become familiar with the S scale

1.

1.1 Tell the students to examine the scale labeled S and observe the numerals. All S scales have a set of numerals that start with 6 and continue through 7, 8, 9, and 10. Thereafter, 15, 20, 25, 30, 40, 50, 60, 70, and 90 are customarily shown. Explain that these numerals represent the measures of angles in degrees.

Some S scales have numerals for the complementary angles also shown beside the same graduation marks. Have the students observe that for these numerals the sequence increases as read from right to left.

1.2 Have the students set the hairline on the graduation for 30 on the S scale, reading right to left. They can then read the corresponding function-value on C. It is 0.5, since $\sin 30^\circ = 0.5$. Have them read other values in this way, and check by means of a table of the sine function. Also, have them find the angles corresponding to given function values, and check with the table. Point out that all function values for this scale are in the range from 0.1 to 1.

1.3 Show the students how to use the S scale in combined operations. For example, to find $3 \sin 30^\circ$ by setting the right-hand index of C over 3 of D, move the hairline over 30 on S and then read 1.5 on D.

1.4 Explain to the students that cosine function-values may be read by using the set of numerals for the complementary measures. Follow the same procedure as was used for the S scale.

1.5 Call attention to the ST scale if there is one on the slide rules available. Have the students observe that this scale has graduations and some numerals for angle measures ranging from 0.57° to 5.7° , and, hence, is a continuation of the regular S scale for smaller angle measures. Show, by methods similar to those suggested above, how it is read and used. Be sure the students know that function values read on C (or D) are all in the range from 0.01 to .1.

**Helping students
become familiar
with the T scale**

2.

2.1 Call attention to the T scale for tangents and help the students become familiar with it. In particular, have them notice it ends with 45° . However, with this scale they can continue finding values of the tangent for angles in the interval 45° to 84.3° by reading the set of numerals on the left of the graduation marks. The angle values increase now from right to left, and the corresponding tangent function-values are read from the CI scale. Have the students first check familiar values, such as $\tan 60^\circ = 1.732$, then check others with a table.

2.2 Explain that the sine function values and tangent function values for small angles are nearly the same, so the ST scale is used for tangents as well as sines, and this explains why it is labeled ST.

Teaching slide rule operation in terms of logarithms

The material in this section should be easy for students who have studied logarithms previously. This section, then, provides suggestions for a theoretical type of introduction to the slide rule. Such a theoretical approach is not, as a rule, wise for the majority of students.

Later some students may be interested in the theoretical principles. These students, and mathematically-talented students generally, will enjoy finding scale formulas as suggested here.

This Teaching Guide does not include comments on log log scales and other more advanced types of scales. These scales are rarely taught in high schools, and when they are, the teacher is undoubtedly a slide rule "fan" who does not need any special suggestions.

When students know what a logarithm is, and also the rules for operation when using logarithms, the basic principles of the slide rule can be explained rather easily.

Helping students understand how the slide rule mechanically adds or subtracts logarithms

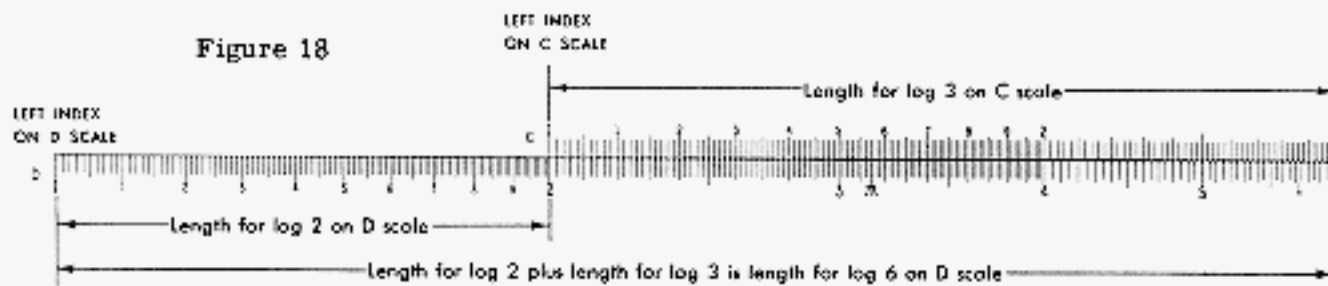
1.

1.1 Review with the students the meaning of the term logarithm (a special name for an exponent) and the definition for base 10. That is, if $10^a = b$, then $a = \log b$.

1.2 Review the two basic rules for operating with logarithms. If $\underline{m} \times \underline{n} = \underline{p}$, then $\log \underline{m} + \log \underline{n} = \log \underline{p}$.
If $\underline{m}/\underline{n} = \underline{q}$, then $\log \underline{m} - \log \underline{n} = \log \underline{q}$.

1.3 Remind the students that the L scale is a uniform scale suitable for measuring. Also, have them recall that it provides logarithms of numbers whose graduations are located on C (or D).

Figure 18



Have the students set the slide to multiply 2×3 using the C and D scales.

They can then observe that one is really adding $\log 2 = .301$ (on L) and $\log 3 = .477$, which is the distance from 1 to 3 on C, and obtains under the hairline the total distance 0.778 on L. Since this distance is the logarithm of 6, one can read the product on D. Have them explain several examples in this way.

1.4 Have the students make some settings for easy divisions (for example, $8 \div 2 = 4$) and explain how the result is obtained by subtracting logarithms mechanically.

Helping students understand the basic scale formulas

2.

2.1 If time permits, introduce the idea of the scale formula. This is a formula for the distances to be measured in graduating a scale. For the D scale, the formula is $\underline{d} = M \log \underline{n}$, where M represents the number of inches or centimeters of scale length to be provided, and \underline{n}_D is the number whose numeral will be placed \underline{d} units from the left index of the D scale. Thus, for a 10-inch scale, $\underline{d} = 10 \log \underline{n}_D$. For $\underline{n} = 3$, $\underline{d} = 10 \log 3 = 10 \times 0.477 = 4.77$ inches. Have the students use a table of logarithms to calculate other values of \underline{d} .

Introduce the scale formula for the A scale; namely $\underline{d} = (M/2) \log \underline{n}_A$, and calculate a few values of \underline{d} .

Finally, show that the hairline equalizes the distances for, say, the D scale and A scale so that

$$\begin{aligned} M \log \underline{n}_D &= (M/2) \log \underline{n}_A \\ \text{or } \log \underline{n}_D &= 1/2 \log \underline{n}_A \\ &= \log \underline{n}_A^{1/2} = \log \underline{n}_A \end{aligned}$$

Hence, $\underline{n}_D = \underline{n}_A$.

In words, this means that for any setting of the hairline, the number on the D scale is the square root of the corresponding number on the A scale.

2.2 Some students may be challenged to construct the scale formulas for other scales and prove the relations that exist between various scales. For example, for the K scale the formula is $\underline{d} = (M/3) \log \underline{n}_K$.